Correct scaling relation for the conserved Kardar-Parisi-Zhang equation and growth models

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To test the corrected scaling relation $\alpha+z=4-3\delta$ of the conserved Kardar-Parisi-Zhang (CKPZ) equation suggested by Janssen [Phys. Rev. Lett. **78**, 1082 (1997)], a stochastic growth model that follows the CKPZ equation with the conserved noise is restudied. We have found a consistent nonzero correction term δ, which is somewhat larger than that estimated from the two-loop renormalization group calculation.
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Recently there has been great interest in kinetic surface roughening phenomena of various growth models [1]. The main quantities of interest are the exponents α, β, and z, which characterize the dynamic scaling law of the surface width W. The surface width $W(L,t)$ is defined by the root mean square fluctuation of the interface height. The dynamic scaling ansatz for $W(L,t)$ in a finite system with the lateral substrate size L is [2]

$$W(L,t)\sim L^{\alpha}f(t/L^{z}), \tag{1}$$

where the scaling function $f(x)$ is x^{β} $(\beta=\alpha/z)$ for $x\ll 1$ and is constant for $x\gg 1$. Among the growth equations that follow the scaling relation (1), the growth equations of the conserved form [3–12] have intensively been studied because of the possible relevance to the real molecular beam epitaxial (MBE) growth. Especially Sun Guo and Grant, Lai and Das Sarma, and Villain have studied the so-called *conserved Kardar-Parisi-Zhang equation* (CKPZ) [3–5]:

$$\frac{\partial h(x,t)}{\partial t}=-\nu_4\nabla^4 h(x,t)+\lambda\nabla^2(\nabla h)^2+\eta(x,t), \tag{2}$$

$$\langle \eta(x,t)\eta(x',t')\rangle=2D(-\nabla^2)^a\delta(x-x')\delta(t-t'). \tag{3}$$

Equation (2) with conserved noise case $(a=1)$ was originally studied by Sun Guo and Grant [3] and that with nonconserved noise $(a=0)$ was suggested for the ideal MBE growth [4,5]. The one-loop renormalization group (RG) calculation showed that the interface roughening described by Eq. (2) with $a=1$ [3] or $a=0$ [4] satisfies the scaling relation

$$\alpha+z=4. \tag{4}$$

However, Janssen [6] has recently claimed that the scaling relation (4) was derived from ill-defined transformation [3,4] and the relation should be modified as

$$\alpha+z=4-3\delta \tag{5}$$

and thus the exponents for Eq. (2) on the d-dimensional substrate with $d<d_c$ are

$$\alpha=\frac{\epsilon}{3}-\delta,\quad z=4-\frac{\epsilon}{3}-2\delta,\quad \beta=\alpha/z\approx\frac{\epsilon}{12-\epsilon}-\gamma, \tag{6}$$

where $\epsilon=d_c-d$ and $\gamma=(36-9\epsilon)\delta/(12-\epsilon)^2$. Here d_c is the uppercritical dimension of the substrate in which the exponents α and β become zero or W depends logarithmically on t and L. $d_c=4$ for the nonconserved noise of $a=0$ and $d_c=2$ for the conserved noise of $a=1$ [6]. From two-loop RG calculation Janseen [6] has shown that the value of δ is very small $(\delta<0.03)$. Recently the corrected scaling relation (5) has been tested [10] by using a stochastic growth model that follows Eq. (2) with the nonconserved noise of $a=0$. It has been found that there must exist the consistent correction term δ and the estimated magnitude of the δ is about 0.05 for $d=2$ or for $\epsilon=d_c-d=2$ [10].

In this paper we test the corrected scaling relation (5) for the conserved noise case of $a=1$ by use of a stochastic growth model that follows Eq. (2) with $a=1$. Because of the smallness of δ, the values of the exponents with $\delta=0$ in Eq. (6) might be very good approximations. However, from a theoretical point of view, it should be very important to find that there exists a nontrivial correction term δ in a growth

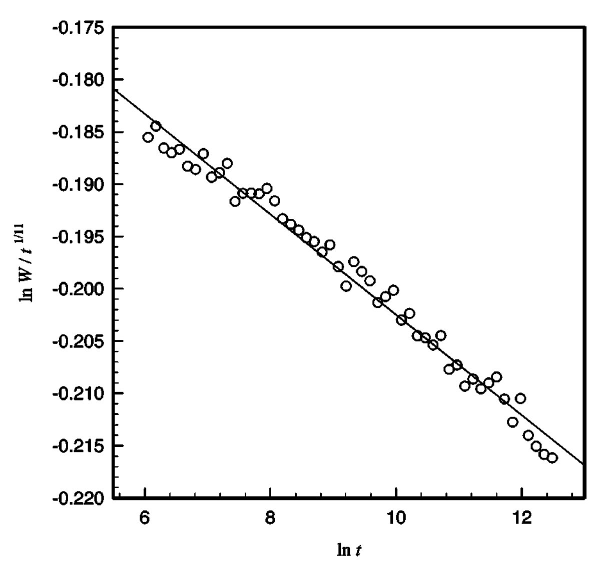

FIG. 1. Plot of $\ln W/t^{1/11}$ against $\ln t$ for $d=1$. Solid line corresponds the line with $\gamma=0.0048$ $(\delta=0.02)$.

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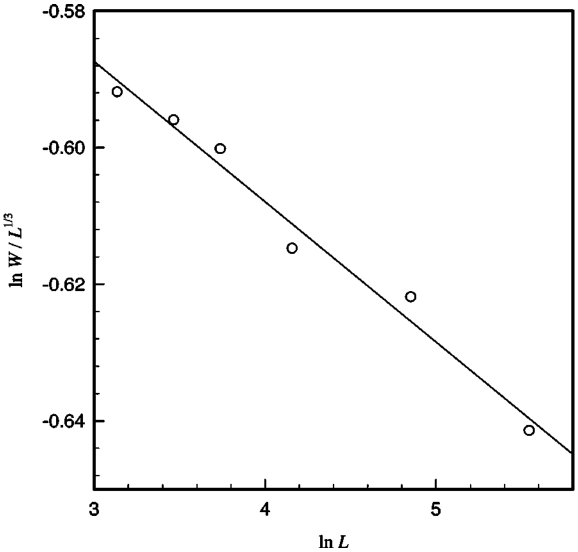

FIG. 2. Plot of $\ln W/L^{1/3}$ against $\ln L$ for the steady-state regime. The slope of the solid line is $(-\delta)\approx -0.02$.

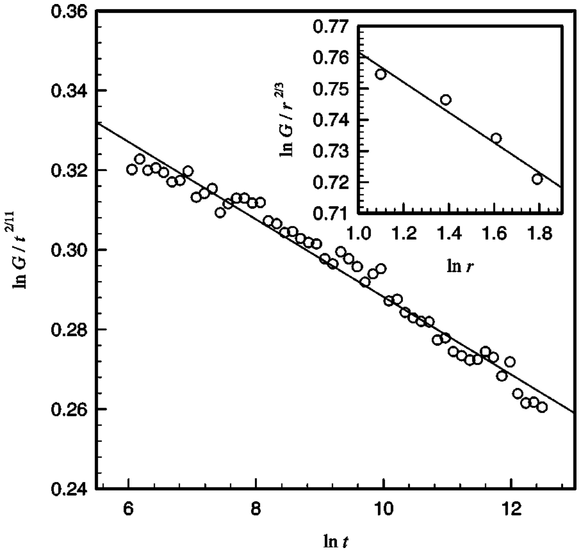

FIG. 3. Plot of $\ln G/t^{2/11}$ against $\ln t$ for $r=395$. The inset shows the plot of $\ln G/r^{2/3}$ against $\ln r$ for $t=264454$.

model with conserved noise as in a model with nonconserved noise [10]. Among the stochastic growth models that are claimed to follow Eq. (2) with $a=1$ [3,7,11], we have chosen the growth model suggested by Krug [11] for the test. The main reason for the choice is that Krug's model has explicitly been shown to satisfy $d_c=2$ [12], which we have also checked. Since d_c of Eq. (2) with $a=1$ is 2 [6], Krug's model is sure to follow CKPZ equation with conserved noise. Krug's model is defined as what follows. Select a column x at random. If the surface configuration satisfies the condition $h(x+1,t)>h(x)$ or $h(x-1,t)>h(x)$, the column x is regarded as immobile and select a new column. If not, the particle at $[x,h(x,t)]$ is moved to a randomly chosen neighboring column.

Since $d_c=2$ for both Krug's model and Eq. (2) with $a=1$, the test of the consistent existence of the term δ in Eq. (5) through the simulation can be done only for $d=1$. In our simulation, we have used the periodic boundary condition. All results are averaged 50 independent runs. The result for the measurement of $W(L,t)$ on a substrate of $L=2\times 10^5$ for $t\ll L^z$ has been displayed in Fig. 1. If $\delta=0$, the exponent β would be 1/11 in $d=1$ as can be seen from Eq. (6). To estimate δ, we have presented the time dependence of $W(L,t)$ through the plot of $\ln W(L,t)/t^{1/11}$ against $\ln t$ in Fig. 1. If $W(L,t)/t^{1/11}$ did not have a consistent time dependence, δ would be estimated 0. Instead $W(L,t)/t^{1/11}$ follows the power law $t^{-\gamma}$ quite well as shown in Fig. 1. From the slope of the fitted line in Fig. 1 we have obtained $(-\gamma)\approx -0.0048$, and thus δ is estimated from Eq. (6) as

$$\delta\approx 0.02. \tag{7}$$

To estimate δ through the exponent α, $W(L,t=\infty)$ on a various substrate size $L=16,23,32,64,128,256$ has been measured in the steady-state regime $t\gg L^z$. If $\delta=0$, the exponent α would be 1/3 in $d=1$ [see Eq. (6)]. To get an estimation of δ from $W(L,\infty)$, we have used the relation $W/L^{1/3}\sim L^{-\delta}$. Figure 2 shows the plot of $\ln W/L^{1/3}$ against $\ln L$. From the slope of the fitted line in Fig. 2, we get nearly the same δ as in Eq. (7). The estimated value of δ from $W(L,t)$ for $t\ll L^z$ is consistent with that from $W(L,\infty)$ and this consistency supports the existence of the nonzero correction term δ.

For another test for the consistent existence of the nonzero correction term δ, we have also investigated the height-height correlation function $G(r,t)\equiv\langle[h(x+r,t)-h(x,t)]^2\rangle$. $G(r,t)$ has the scaling property $G(r,t)=r^{2\alpha}g(r/t^{1/z})$ [11], where $g(x)\simeq x^{-2\alpha}$ for $x\gg 1$ and $g(x)\simeq\text{const}$ for $x\ll 1$. For a given r with the condition $r\gg t^{1/z}$, $G(r,t)\simeq t^{2\beta}$. Figure 3 shows the plot of $\ln G/t^{2/11}$ against $\ln t$ for $r=395$. Using the relation $G/t^{2/11}\simeq t^{-2\gamma}$, we have obtained $(-2\gamma)\approx -0.0097$, and $\delta\approx 0.02$. The inset of Fig. 3 shows the plot of $\ln G/r^{2/3}$ against $\ln r$ for $r=3,4,5,6$ at time $t=264\,454$. Using the relation $G/r^{2/3}\simeq r^{-2\delta}$, we have estimated $\delta\approx 0.02$. δ's obtained from $G(r,t)$ are also very close to the values obtained from Figs. 1 and 2.

We have summarized the estimated δ's for Eq. (2) with conserved noise ($a=1$) and that with nonconserved noise ($a=0$) [10] in Table I. In both cases there exist consistent nonzero correction terms. The estimated δ's are larger than Janssen's value [6]. Since RG calculation is based on an

TABLE I. Estimated δ's for Eq. (2) with nonconserved or conserved noise from two-loop RG ϵ-expansion [6] and the growth models.

	Two-loop RG ϵ expansion	Growth models
Conserved noise ($d=1$)	0.0025	0.02 [11]
Nonconserved noise ($d=2$)	0.014	0.05 [9]

asymptotic ϵ expansion, the estimated δ only for small ϵ is quantitatively meaningful as one can see from RG ϵ expansion for the ordinary n-vector model of magnetism [13] and thus quantitative comparison of δ to the simulation results might not be so important. Rather it is more important to establish the consistent existence of the term δ.

In summary, we have tested the corrected scaling relation (5) for Eq. (2) with conserved noise ($a=1$) by use of Krug's model and found a consistent nonzero correction term δ, which is somewhat larger than the value of δ estimated from RG two-loop ϵ-expansion.

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[1] J. Krug and H. Spohn, in *Solids Far From Equilibrium: Growth, Morphology and Defects*, edited by C. Godreche (Cambridge University Press, New York, 1991); *Dynamics of Fractal Surfaces*, edited by F. Family and T. Vicsek (World Scientific, Singapore, 1991); A.-L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, England, 1995); T. Halpin-Healy and Y. -C. Zhang, Phys. Rep. **254**, 215 (1995).

[2] F. Family and T. Vicsek, J. Phys. A **18**, L75 (1985).

[3] T. Sun, H. Guo, and M. Grant, Phys. Rev. A **40**, 6763 (1989).

[4] Z. W. Lai and S. Das Sarma, Phys. Rev. Lett. **66**, 2348 (1991); L. H. Tang and T. Nattermann, *ibid.* **66**, 2899 (1991).

[5] J. Villain, J. Phys. I **1**, 19 (1991).

[6] H. K. Janssen, Phys. Rev. Lett. **78**, 1082 (1997).

[7] Z. Rácz, M. Siegert, D. Liu, and M. Plischke, Phys. Rev. A **43**, 5275 (1991).

[8] D. E. Wolf and J. Villain, Europhys. Lett. **13**, 389 (1990).

[9] S. Das Sarma and P. I. Tamborenea, Phys. Rev. Lett. **66**, 325 (1991).

[10] S. H. Yook, J. M. Kim, and Yup Kim, Phys. Rev. E **56**, 4085 (1997).

[11] J. Krug, Adv. Phys. **46**, 139 (1997).

[12] I.-M. Kim, J. Yang, and J. M. Kim, J. Korean Phys. Soc. **31**, 898 (1997).

[13] *Phase Transition and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic Press, London, 1976), Vol. 6.