Origin of the log-normal popularity distribution of trending memes in social networks

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We study the origin of the log-normal popularity distribution of trending memes observed in many real social networks. Based on a biological analogy, we introduce a fitness of each meme, which is a natural assumption based on sociological reasons. From numerical simulations, we find that the relative popularity distribution of the trending memes becomes a log-normal distribution when the fitness of the meme increases exponentially. On the other hand, if the fitness grows slowly, then the distribution significantly deviates from the log-normal distribution. This indicates that the fast growth of fitness is the necessary condition for the trending meme. Furthermore, we also show that the popularity of the trending topic grows linearly. These results provide a clue to understand long-lasting questions, such as what causes some memes to become extremely popular and how such memes are exposed to the public much longer than others.

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I. INTRODUCTION

On-line social media grows at an explosive rate. Through online social media hundreds of millions of people are exposed to different cultural entities such as ideas, news, and technology. Such a cultural entity is called a meme. The term meme was first coined by Dawkins to represent a cultural analogy with genes [1]. When such cultural entities or memes spread over a society, they evolve via replication and mutation in human culture [2], which resembles the evolution of genes in biological environments. Uncovering various dynamical and statistical properties of meme spreading is very important to understand many social phenomena and is also crucial for possible applications. For example, identifying influential nodes and understanding their dynamical behavior in social networks [3,4] can be effectively applied to viral marketing and it is closely related to interesting physical processes or phenomena such as the avalanche process and critical phenomena in statistical mechanics.

Besides replication and mutation, memes also compete with each other to get our attention. Only a small number of memes can survive and acquire a large popularity through the competition, but most memes disappear without attracting much attention. Therefore, how and why only a small number of memes can survive and acquire a large popularity are very interesting questions in social phenomena. Recent available big data produced by various online social media have provided some clue to understand the intriguing phenomena [4–8]. In these studies, the popularity of the meme is generally measured as the total number of times the specific meme is exposed (or the total number of reposts) through online social media. Based on the empirical data analysis, it has been shown that the meme popularity distribution is well described by a heavy-tailed or power-law distribution, which resembles critical phenomena in statistical physics. Possible origins of the criticality have been suggested to be the limited attention and underlying topology [9], or competition-induced criticality [10]. More recently, it has been uncovered that the criticality in meme population distribution is originated from the balance between innovation of a new meme and extinction of old memes. The resulting power-law distribution is quite robust, regardless of the underlying structure [11].

On the other hand, the relative popularity distributions of the trending topics of Twitter [12] and digg [13] have been investigated. Here, the trending topics mean the memes of large popularity at a given time. Unlike the distribution of popularity for all memes, the relative popularity distribution of trending topics is well approximated by the *log-normal* distribution [14,15]. Since the central limit theorem states that the probability of the sum of independent and identically distributed random variables becomes Gaussian, the logarithm of relative popularity obtained from the multiplicative process can be described by a log-normal distribution. Furthermore, the empirical data show that the (relative) popularity grows linearly in time but the growth is eventually curtailed, which suggests that the interest in a specific topic decays as time increases. Thus, stochastic models based on the multiplicative process with decaying novelty were suggested to explain the empirically observed behavior of the relative popularity for trending memes [14,15].

However, the log-normal distribution is observed only for the trending memes, which is still in contrast with the fat-tailed or power-law distribution obtained from all memes. Furthermore, it is still not clearly understood what causes some memes to become extremely popular, or how such topics are exposed to the public longer than others. A study on the contextual content of memes showed that the uniqueness of memes is crucial for the occurrence of long-lasting memes with large popularity [16]. Thus, it is natural to assume that each meme has a different level of interest depending on its contents. For example, people have a high interest in the political policies of the U.S., but only a few people have an interest in the lunch menu of the cafeteria in a university.

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The existence of such different levels of interest in memes indicates that each meme can have its own quality value. The interplay among the intrinsic quality, the limited attention, the innovation rate, and the meme popularity and diversity was studied [17]. The level of interest of each meme in a social environment resembles the fitness of a species or a gene in biological systems [18]. Therefore, in order to investigate the possible origin of the emergence of trending memes with extreme popularity and to study the origin of the log-normal distribution of relative popularity of trending memes, we introduce a generalized meme competition model with fitness. From the numerical simulations of the model, we show that the time dependent fitness plays an important role for the emergence of high popularity memes. The time dependence of the fitness reflects the inclination that the trending meme draws more attention from society as time goes on. Moreover, the model also clearly provides the condition for the log-normal relative popularity distribution of trending memes and the linear growth of popularity, which is observed in the empirical data [14,15].

II. MODEL

In order to investigate the origin of the log-normal distribution of relative popularity for trending memes, we introduce a meme propagation model with fitness (MPMF). The MPMF is similar to the competition-induced criticality model suggested by Gleeson *et al.* [10], but each meme has its own fitness in the model. In the MPMF each node (or agent) has a screen. The screen has the capacity for only one meme, and a meme of the current interest of the node is displayed on the screen. At $t = 0$, we start from a network of N nodes. Every meme α at the time t has its own fitness $f_\alpha(t)$. In the model, either the propagation process or the innovation process is taken at each time step. In *the propagation process*, a node i is selected with the probability

$$\Pi_i(t) = \frac{f_i(t)}{\sum_{j=1}^{N} f_j(t)}, \tag{1}$$

where $f_i(t)$ is the fitness of the meme on the node i. Then, the meme on i propagates to all k_i connected nodes to i, where k_i is the degree of node i. When the meme propagates to k_i connected neighbors, the memes on the screens of connected neighbors are replaced by the meme of node i with $f_i(t)$. And the popularity of the meme increases by 1. In *the innovation process*, a new meme with a fitness f^* is generated on a randomly selected node.

Since the topological features of many social networks are characterized by the concepts of "small-world" and "scale-free" [19], we assume that the underlying topology is a scale-free network (SFN). Even though real social networks are directed in general, it has already been shown that the directedness does not affect the main result [11]. Therefore, we use undirected SFNs to represent the structure of underlying social networks. A SFN is characterized by the power-law degree distribution, $P(k) \sim k^{-\gamma}$. To generate the SFN with tunable γ, we use a static SFN model introduced by Goh *et al.* [20]. All the quantities are averaged over 100 network realizations and 1000 independent runs on each network. We mainly use the SFNs with $N = 10^4$, $\langle k \rangle = 10.4$, and $\gamma = 2.5$ in the subsequent simulation studies. We check that the main conclusions of our paper are nearly identical regardless of N, $\langle k \rangle$, or γ. Furthermore, we confirm that a different underlying topology like the small-world networks [21] does not change the results as presented in Appendix A.

III. THE BEHAVIOR OF TRENDING MEMES

In order to investigate the origin of the log-normal distribution of relative popularity for trending memes [14,15], we consider our model under the following conditions. Initially, we set the fitness of the meme on each node to be $f_i(0) = 1$. At $t = 0$, we take an innovation process, in which a meme with the fitness $f^*(0)$ ($\geqslant 1$) is generated on a randomly selected node j. The fitness $f^*(t)$ evolves as

$$f^*(t) = F(t) f^*(t-1), \tag{2}$$

where $F(t)$ is some function that determines the time evolution of the fitness of the meme. The functional form will be discussed later. Equation (2) reflects the inclination that the trending meme draws more attention as time goes on. On the other hand, the fitness of the other meme remains at a constant, $f(t) = 1$, for $t \geqslant 0$. Thus, only the meme with $f^*(t)$ is a special meme which can evolve into a trending meme, while the other memes play the role of background memes with average fitness. Since we are interested only in the relative popularity for the trending memes, we focus on the relative popularity of the meme with fitness $f^*(t)$.

The innovation of the background memes with average fitness [or $f(t) = 1$] does not change $\Pi_i(t)$ in Eq. (1) for propagation of the meme with $f^*(t)$ if the innovation of a background meme would not change its fitness. Furthermore, the types of background memes with average fitness [or $f(t) = 1$] do not affect $\Pi_i(t)$ for the propagation of the special meme or the relative popularity of the meme with $f^*(t)$. In addition, due to the relatively rare occurrence of the meme with large $f^*(0)$, we only take the propagation processes for $t > 0$.

The popularity of the innovated meme at t, $n(t)$, is defined as the cumulative number of propagations (or repostings) up to t. The relative popularity of the innovated meme, $C(t_2, t_1)$, ($t_2 > t_1$), is defined as [14]

$$C(t_2, t_1) = n(t_2)/n(t_1). \tag{3}$$

To find how popularity of a certain meme grows in time and finally becomes a trending meme, we only consider the case that $f^*(t)$ grows during the considered time period. The popularity is measured only for the memes which survive after a given initial transient period ($t > 50$). We obtain the relative popularity distribution, $P(C)$, from the 10^5 survival samples by using 100 network realizations and 1000 independent runs on each network. In Fig. 1, $P(C)$, the probability distribution of $C(t_2, t_1)$ of the meme with $f^*(0) = 1$ and $F(t) = A$ [$(A > 1)$] measured from simulations is shown. If the propagation occurs n times then $f^*(t+n) = A^n f^*(t)$. In Fig. 1 we display the measured $P(C)$s with $A = 1.20$, as an example. The obtained $P(C)$ satisfies the log-normal distribution

$$P(C) = \frac{1}{C\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(\ln C - \ln C_0)^2}{2\sigma^2}\right\}. \tag{4}$$

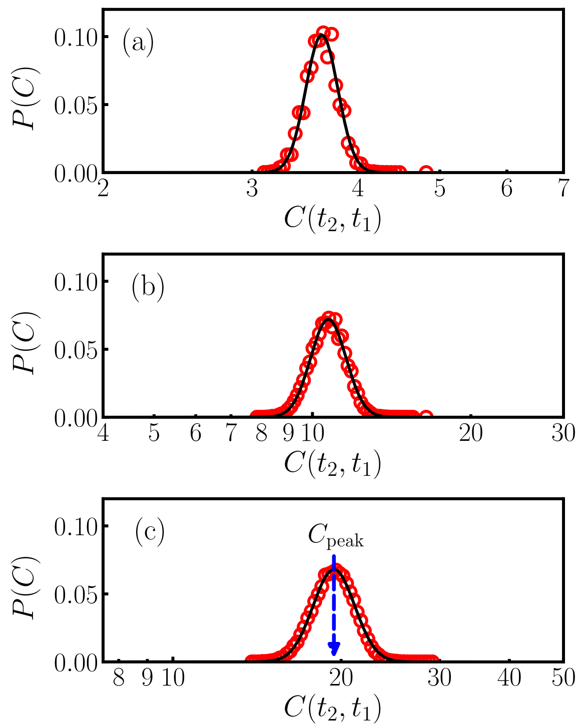

FIG. 1. Plot of $P(C)$'s for the meme with $f^*(0) = 1$ and $F(t) = A = 1.20$ for (a) $C(200, 100)$, (b) $C(700, 100)$, and (c) $C(1500, 100)$. Curves denote the log-normal function (4) with (a) $C_0 = 3.6$ and $\sigma = 0.04$, (b) $C_0 = 10.8$ and $\sigma = 0.08$, and (c) $C_0 = 19.6$ and $\sigma = 0.09$.

For example, we obtain $C_0 \simeq 3.6$ and $\sigma \simeq 0.04$ for $C(200, 100)$ [Fig. 1(a)], $C_0 \simeq 10.8$ and $\sigma \simeq 0.08$ for $C(700, 100)$ [Fig. 1(b)], and $C_0 \simeq 19.6$ and $\sigma \simeq 0.09$ for $C(1500, 100)$ [Fig. 1(c)] from the best fit of the data to Eq. (4). The results for the meme with exponentially growing fitness clearly explain the observed log-normal distribution of trending memes in a real social network [14]. As we increase t_2/t_1, we find that the peak of $P(C)$ decreases, and the distribution becomes broader. As the ratio t_2/t_1 becomes larger, some memes with fitness f^* disappear due to the accumulated effects of the propagation of the background memes. Thus, the height of the peak decreases as t_2/t_1 increases.

In Figs. 2(a) and 2(b) we display $P(C)$ for the meme with $f^*(0) = 100$ and 10 000, and $A = 1.2$, respectively. The data in Figs. 2(a) and 2(b) clearly show that the width of the peak of $P(C)$ becomes narrow as $f^*(0)$ increases. This can be easily understood from the limiting case of $f^*(0) \to \infty$. In this limit, only the innovated (trending) meme with $f^*(t)$ is selected to propagate to the connected neighbors at each time step. Thus, the popularity, $n(t)$, of the innovated meme grows as $n(t) \propto n(0)t$. Since $n(t)$ grows linearly, the relative popularity of the meme grows as $C(t_2, t_1) = n(t_2)/n(t_2) \sim t_2/t_1$. Thus, $P(C)$ for the meme with high fitness $f^*(0)$ has a delta-function-like peak at $C_{\rm peak}$ when $f^*(0)$ is sufficiently large as shown in Figs. 2(a) and 2(b), and $C_{\rm peak}$ scales as $C_{\rm peak} \sim t_2/t_1$ [see Fig. 2(c)]. The linear growth of the popularity of the trending meme has been observed in the empirical data [14]. Therefore, these results strongly suggest that the linearly growing popularity of trending memes [14] is originated from the exponentially growing fitness of the specific memes. This also clearly shows why only a few memes can acquire large popularity and can survive longer than other memes. Various forms of $F(t)$ in Eq. (2) are also tested and we obtain identical results when $F(t) > 1$ for any t.

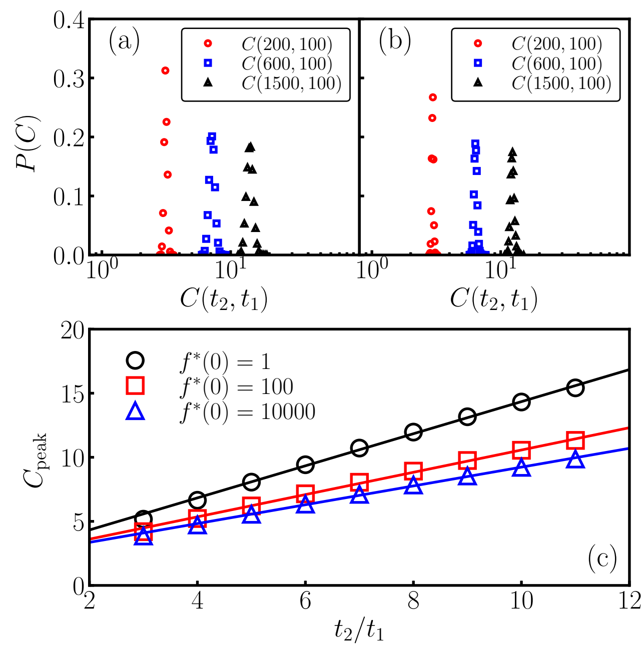

FIG. 2. Plots of $P(C)$ for the meme with (a) $f^*(0) = 100$ and (b) $f^*(0) = 10\,000$. (c) Plots of $C_{\rm peak}(t_2, t_1)$ against t_2/t_1 with $t_1 = 100$. The solid lines represent the relation $C_{\rm peak}(t_2, t_1) \sim t_2$.

Note that in this model we obtain the log-normal distribution and linear growth of the popularity by assuming the exponential growth of the fitness of a specific meme, unlike the previous studies [14,15] in which the multiplicative process with decaying novelty was suggested as the main mechanism for the log-normal distribution of the popularity. Furthermore, such log-normal distribution of the meme popularity is observed only for the trending memes. The popularity distribution for all memes in a social network is known to be a heavy-tailed distribution [10,11]. Thus, our results indicate that the meme with exponentially growing fitness more easily becomes a trending one, and is one possible origin of log-normal distribution of relative popularity of trending memes.

For a systematic investigation of the characteristic feature of the trending meme, we increase $f^*(t)$ with

$$F(t) = \begin{cases} 1 + t & \text{if } t/m = \text{natural number} \\ 1 & \text{otherwise.} \end{cases} \tag{5}$$

Here m is a natural number. Thus $f^*(t)$ changes its value at every mth update. Equation (5) interpolates the behavior between the trending meme and nontrending meme. When $f^*(0)$ is small and $m \to \infty$, $f^*(t)$ never increases, which corresponds to the case of a nontrending meme. $P(C)$ for $C(2000, 500)$ of the meme with $f^*(0) = 2$ when $m \to \infty$ is displayed in Fig. 3(a), as an example for nontrending meme. As shown in Fig. 3(a), if f^* does not grow and is comparable to the fitness $f_i (= 1)$ of background memes, we find that $P(C)$ significantly deviates from the log-normal distribution and

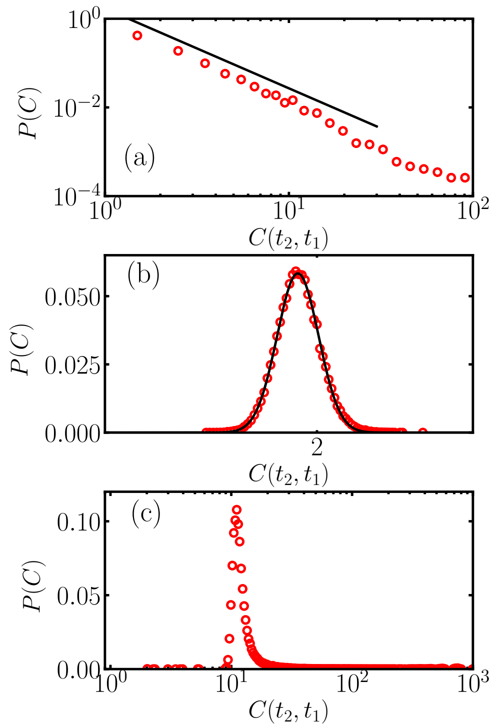

FIG. 3. Plot of $P(C)$ for $F(t) = 1 + t$ with (a) $f^*(0) = 2$, $m \to \infty$ when ($t_1 = 500, t_2 = 2000$), (b) $f^*(0) = 1$, $m = 5$, when ($t_1 = 700, t_2 = 2000$), and (c) $f^*(0) = 100$, $m = 100$, when ($t_1 = 300, t_2 = 4000$). The solid line in (a) represents the relation $P(C) \sim C^{-\delta}$ with $\delta = 1.8$ and that in (b) represents the log-normal distribution with $C_0 = 1.94$ and $\sigma = 0.03$.

becomes a heavy-tailed distribution. In this case, $P(C)$ is well fitted to the power law

$$P(C) \sim C^{-\delta}, \tag{6}$$

with $\delta \approx 1.8 \pm 0.2$. Other values of t_2 show nearly the same behavior as in Fig. 3(a) if f^* is sufficiently small and comparable to that of background memes. This behavior of $P(C)$ in Fig. 3(a) implies that all memes including the innovated one compete with each other before gaining large popularity. Furthermore, the obtained value of δ is very similar to that for the popularity distribution of a meme having the same fitness in Ref. [11]. If $f^*(0)$ is not much larger than $f_i(= 1)$, then the fitnesses of all memes on the networks are nearly identical. As a result, the innovated meme cannot become a trending meme. Furthermore, when $t_1 \to 0$, then $n(t_1)$ for a meme with f^* approaches $n(t_1) \to \mathcal{O}(1)$ and $C(t_2, t_1) \simeq n(t_2)$. Thus, $P(C)$ for the meme with f^* should show the same behavior as the cumulative popularity distribution, $P(n)$, for all memes with equal fitness. As shown in Ref. [11], $P(n)$ for an early transient period is known to show a heavy-tailed distribution when there is no innovation process. Therefore, $P(C)$ for a meme with f^* is well fitted to the power law (6).

On the other hand, when $f^*(0)$ is small but increases by Eq. (5) with $m > 1$, the obtained $P(C)$s are well approximated by the log-normal distribution. The obtained $P(C)$ for $f^*(0) = 1$ and $m = 5$ is shown in Fig. 3(b), which provides more evidence that the meme the fitness of which increases exponentially becomes the trending meme. Thus, Eq. (5) successfully interpolates the behavior between the trending and nontrending memes.

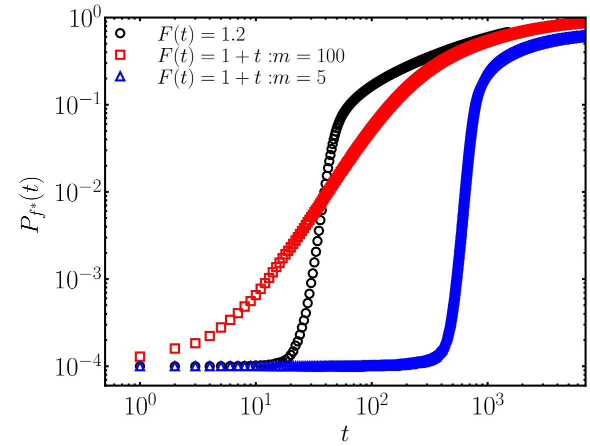

FIG. 4. Plot of $P_{f^*}(t)$ for the meme for $f^*(0) = 1$ with $F(t) = 1.2 =$ const (black circles), $f^*(0) = 100$ with $F(t) = (1 + t)$ and $m = 100$ (red squares), and $f^*(0) = 1$ with $F(t) = (1 + t)$ and $m = 5$ (blue triangles).

Furthermore, when $f^*(t)$ becomes much larger than fitness of the background, $P(C)$ deviates from the log-normal distribution, even though the meme with $f^*(t)$ becomes a trending one. In Fig. 3(c) we display the obtained $P(C)$ for $f^*(0) = 100$ and $m = 100$. The data in Fig. 3(c) clearly show that $P(C)$ deviates from the log-normal distribution, even though $P(C)$ has a peak.

In order to investigate the different behaviors between the memes with small and large $f^*(t)$, we measure the fraction $P_{f^*}(t)$ of the nodes which have the meme with f^*. When $P(C)$ of the trending meme becomes log normal, $P_{f^*}(t)$ shows a characteristic behavior. As shown in Fig. 4, $P_{f^*}(t)$ remains at the value of $1/N$ for a relatively long period. Then it abruptly increases and approaches $P_{f^*}(t) = 1$ for $F(t) =$ const and $1 + t$ with $m = 5$. In contrast, $P_{f^*}(t)$ for $F(t) = 1 + t$ with $m = 100$ shows a clearly different behavior from that for the case of the log-normal distribution. $P_{f^*}(t)$ for $F(t) = 1 + t$ with $m = 100$ increases continuously and smoothly. We also measure $P_{f^*}(t)$ for various N and find that the behavior of $P_{f^*}(t, N)$ is not affected by N (see Appendix B). Thus the abrupt increase is responsible for the origin of the log-normal distribution of $P(C)$. This resembles the adoption dynamics in various systems having innovation. In such systems, a new technology, genotype, or phenotype emerges and lurks in the background for a relatively long time. Then, it suddenly spreads over the whole system and becomes the most popular one [22–24]. In our model the trending meme also shows a similar property with such adoption dynamics.

IV. SUMMARY AND DISCUSSION

In summary, we study the origin of the log-normal popularity distribution of the trending memes observed in many real social networks. From a biological analogy, we introduce a fitness of each meme, which is a natural assumption based on sociological reasons. From numerical analysis we find that the relative popularity distribution of innovated (trending) memes is well described by the log-normal distribution when the fitness of the innovated meme grows exponentially in time. In addition, we also show that the relative popularity $C(t_2, t_1)$ scales as $C(t_2, t_1) \sim t_2/t_1$. This implies that the popularity of such trending memes grows linearly in time. The obtained results from our model with exponentially growing fitness agree very well with the empirical results reported in Refs. [14,15]. Even though the multiplicative process with decaying novelty was suggested as the main mechanism for the log-normal distribution of the relative popularity, the suggested models cannot provide a complete picture for the evolutionary dynamics of all memes, because the log-normal distribution is observed only for the trending memes. In contrast to the previous models, our model focuses on the fitness of each meme and shows that the exponentially growing fitness is another important possible origin of the log-normal distribution of the popularity. Furthermore, our model also suggests that the fraction of nodes with such trending memes remains at small constant value for a relatively long period. Then, the trending meme abruptly spreads over the entire system like in the many innovation-propagation models [22–24]. This abrupt increase is the main origin of the log-normal distribution and the linear increase of $C(t_2, t_1)$ for the trending meme. As a result, the trending memes are exposed to the public much longer than others.

On the other hand, if the fitness of the innovated meme becomes relatively small and does not grow, then the fitness difference does not affect the popularity distribution. Thus, the popularity distribution can be well described by the power law. This clearly shows that the fitness plays a very important role to acquire a large popularity and its observed distribution. Therefore, our model provides a clue to understand why only a small number of memes can acquire a large popularity in many real social networks, and why such trending memes are exposed to the public longer than others.

Furthermore, the initial fitness of memes generally cannot be so large even for the trending meme and even though they are generated by, for example, mass media or famous organizations. If the fitness of such a meme is extremely large, then the meme would be shared by all users in a social network in a very short time. However, in the real world, such an extreme case is never observed. Therefore, in real social networks, it is more natural that the fitness of the trending meme has a moderate value at its birth than there is some meme the fitness of which is extremely large, and it evolves into some larger value through the interaction with the environments as in biological systems.

In the real world, the number of memes which become trending memes is very small. For example, on Facebook, among hundreds of millions of daily postings (or memes) only a limited number of memes can be trending memes. In addition, in real online social networks, multiple memes can be displayed on the same screen at the same time. Thus each trending meme can behave as an independent meme. Furthermore, due to the limited attention of each user, the memes with large popularity are not shared by all users in the network. Therefore, competition between trending memes should rarely occur, and it is natural to assume that there is no competition between them when the popularity of the meme rapidly grows. Thus, the model studied in this paper is a suitable model to explain the characteristic features of trending memes.

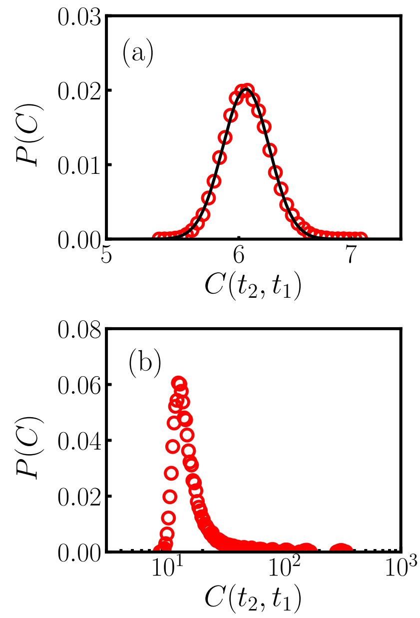

FIG. 5. Plot of P_C's for $F(t) = 1 + t$ with (a) $f^*(0) = 1$, $m = 2$ when ($t_1 = 1000$, $t_2 = 4000$) and (b) $f^*(0) = 1000$, $m = 1000$ when ($t_1 = 50$, $t_2 = 500$). The size of network is $N = 10\,000$. The solid line represents the log-normal distribution with $C_0 = 6.06$ and $\sigma = 0.03$.

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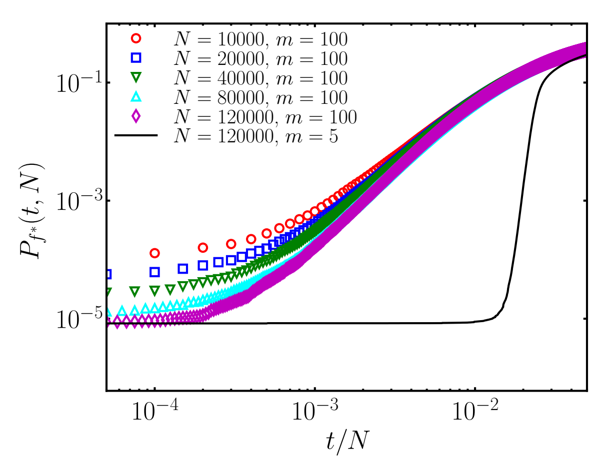

FIG. 6. Plot of $P_{f^*}(t, N)$ of the meme with $f^*(t)$ for various N on SFNs. Symbols represent $P_{f^*}(t, N)$'s of the meme having $f^*(0) = 100$ with $F(t) = (1 + t)$ and $m = 100$. The line represents $P_{f^*}(t, N)$ for $f^*(0) = 1$ with $F(t) = (1 + t)$ and $m = 5$ for $N = 120\,000$.

APPENDIX A: $P(C)$ ON SMALL-WORLD NETWORKS

In Fig. 5 we display $P(C)$s measured on small-world networks (SWNs) with $\langle k\rangle = 10$ and rewiring probability $\phi = 0.1$ [21]. As in Fig. 3(b), when $f^*(0)$ is small but increases by Eq. (5) with $m > 1$, the obtained $P(C)$ is well approximated by the log-normal distribution as in Fig. 5(a). On the other hand, if $f^*(0)$ becomes much larger than fitness of the background memes, $P(C)$ of the trending meme on SWNs significantly deviates from the log-normal distribution as shown in Fig. 5(b). The results agree very well with the measured $P(C)$s on SFNs. Since SWNs are regarded as the scale-free network with $\gamma \to \infty$, the results clearly show that $P(C)$s are not affected by the underlying topology.

APPENDIX B: $P_{f^*}(t, N)$

In Fig. 6 we display $P_{f^*}(t, N)$ for various values of N(= 10 000, 20 000, 40 000, 80 000, 120 000) to show how the size of network affects the behavior of $P_{f^*}(t, N)$. The symbols represent the measured $P_{f^*}(t, N)$ when $F(t)$ is given by Eq. (5) with $f^*(0) = 100$ and $m = 100$. The line in Fig. 6 shows $P_{f^*}(t, N = 120\,000)$ when $f^*(0) = 1$ and $m = 5$ as in Fig. 4. When t is small, $P_{f^*}(t, N)$ for $F(t) = 1 + t$ with $m = 100$ decreases as N increases. Even though t^*/N at which $P_{f^*}(t, N)$ deviates from $1/N$ slightly increases as N increases, $P_{f^*}(t, N)$ for $m = 5$ (black line) stays at the value $1/N$ for a much longer period than that for $m = 100$ as shown in Fig. 6. This clearly shows that the gradual growth of $P_{f^*}(t)$ for $m = 100$ in Fig. 4 is not originated from the finite-size effect.

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