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Non-equilibrium stochastic model for stock exchange market

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HIGHLIGHTS

- The effect of the industrial relationship (IR) on financial market is studied.
- We model the financial market based on the behavior of technical traders.
- We measure the return distribution and autocorrelation function of volatility.
- The heterogeneous IR topology is a possible origin of the universal features.

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ABSTRACT

We study the effect of the topology of industrial relationship (IR) between the companies in a stock exchange market on the universal features in the market. For this we propose a stochastic model for stock exchange markets based on the behavior of technical traders. From the numerical simulations we measure the return distribution, $P(R)$, and the autocorrelation function of the volatility, $C(T)$, and find that the observed universal features in real financial markets are originated from the heterogeneity of IR network topology. Moreover, the heterogeneous IR topology can also explain Zipf–Pareto's law for the distribution of market value of equity in the real stock exchange markets.

1. Introduction

Financial markets have been increasingly investigated in statistical physics as a part of complex systems [1–3]. Empirical studies based on various financial market data have revealed very interesting universal properties which govern the dynamical properties of financial markets. For example, the probability distribution of the price or the market index return, and the autocorrelation function of the volatility have revealed interesting universal features in financial markets [3–5]. Such universal features are usually called as the stylized facts. Thus, uncovering the origin of universal behaviors observed in various financial markets is important to understand the interesting behaviors in financial markets. For this, many stochastic and agent-based models have been proposed [2,6–13].

Since there are no apparently tunable external parameters in financial markets, some studies have focused on the self-organized criticality of financial systems [14,15]. The self-organizing features are frequently observed in various socio- and econo-systems [16,17] as well as biological [18] and technological systems [19] in which cascade or avalanche plays a crucial role. The self-organized criticality is usually characterized by the punctuated equilibrium [20], in which the time series show intermittent occurrences of large bursts separated by relatively long periods of quiescence. In financial time series, the price return is one of the well-known examples for such intermittent occurrences of large bursts. This usually causes the volatility clustering [4] which plays a very crucial role in modeling financial markets.

In this paper, we propose a simple stochastic model which incorporates the behavior of agents in financial markets. In the real market, there are two types of investors. The first are fundamentalists who attempt to determine the fundamental values of stocks. The second are technical traders who make their trading decisions based on the price pattern. Although

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the fundamentalists hold a majority of the stocks, the contribution of the technical traders to the market dynamics is much larger than that of the fundamentalists because of the frequent trading activity of the technical traders [11]. Since the trading decisions of technical traders are based on the price pattern, they can make somewhat coherent decisions. Such coherent behavior of agents is usually called a herd behavior [12,21,22]. Recently we showed that the herd behavior of the agents in a financial market causes the power-law scaling of tails in return distribution and is also related to the economic crisis [22]. In this paper we study the effects of the coarse grained behavior of technical traders and the topology of the industrial relationship (IR) between companies in financial markets on the market dynamics. As we shall show, one possible origin of the observed stylized facts in stock exchange markets is the topology of underlying IR network between the listed companies in stock exchange markets. To verify this we use four different underlying structures, 1*D* and 2*D* regular lattices, random networks, and scale-free networks.

2. Price update rule

The herd behavior of technical traders is frequently observed from the result of coherent decisions in a group of agents to buy (or sell) stocks of a specific company [22]. However, some of the agents in the group would not want to buy (or sell) the stocks if the price of the stock becomes higher (or lower) than their expectations. In such a case, those agents would seek other companies to buy (or sell) stocks. The possible alternatives are the companies which have industrial relationship to the selected company. For example, the prices of stocks of oil companies usually increase or decrease together. Thus, if the stock price of Exxon Mobile becomes too high (low), then some agents would seek other oil companies to buy (sell) their stocks. To incorporate such features into the model we define the following stochastic procedures for stock exchange markets.

Let N be the number of companies listed in a stock exchange market. The price of a stock of ith company at time t is denoted by $p_i(t)$. $p_i(t)$ evolves initially from the same scaled prices $p_i(0) = p_0$ for all i. Here p_0 is a constant and set to be zero in the following simulations. The price is updated by the following steps. (i) Select a company i at random. (ii-1) Increase the price of company i by random increment $\Delta(t)(\in (0, 1])$ with a probability $P\ (p_i(t) \to p_i(t) + \Delta(t))$. This mimics the increase of stock price by a buying decision of a group of agents. (ii-2) Then check the price differences between i and all of its nearest neighbors, j's. (ii-3) For a preassigned positive constant C, if $p_i(t) - p_j(t) \geq C$, then adjust the price of stocks of neighboring companies j upwards $(p_j(t) \to p_j(t) + \Delta(t))$. This process reflects the fact that the price of i is too high to buy; thus the agents would seek other companies which have industrial relationship to the company i with underestimated price. (ii-4) Repeat the processes (ii-2)–(ii-3) for all nearest neighbors of j's until prices of all companies and their nearest neighbors satisfy $p_i(t) - p_j(t) < C$. (iii-1) With the probability $1 - P p_i(t)$ decreases by $\Delta(t)$ $(p_i(t) \to p_i(t) - \Delta(t))$ if $p_i(t) - \Delta(t) \geq 0$. The imposed condition $p_i(t) - \Delta(t) \geq 0$ implies that the price should be positive and provides the temporal minimum of $p_i(t)$. Thus, the agent would not sell their stocks when $p_i(t)$ is equal to the temporal minimum, because they would expect that the stock price of i is underestimated and will increase in near future. (iii-2) Calculate $p_j(t) - p_i(t)$ for all its nearest neighbors j's. (iii-3) If $p_j(t) - \Delta(t) > 0$ and $p_j(t) - p_i(t) \geq C$, then adjust $p_j(t)$ downwards $(p_j(t) \to p_j(t) - \Delta(t))$. (iii-4) Repeat (iii-2)–(iii-3) until all prices of companies satisfy $p_j(t) - p_i(t) \geq C$. This corresponds to the decrease of stock price by the coherent selling decisions. The processes (ii-4) and (iii-4) cause the successive updates of $p_i(t)$'s, which are called as an avalanche [20].

This update rule is reminiscent of Senppen's model in the interface roughening phenomena [23] or innovation propagation model in sociophysics [24]. Since the value of C only affects the scale of price, we set $C = 1$ in the following simulations for simplicity. We also set $P = 1/2$, because if $P < 1/2$, then the average price decreases and when $P > 1/2$ the average price linearly increases as t. The unit time is defined as the usual Monte Carlo time step. For the direct comparison with real market indices, we use $N = 1024$.

3. Underlying structures

To study the effect of the structure of IR on market properties, we consider two different IR network topologies, random network (RN) and scale-free network (SFN), as well as 1-dimensional (1*D*) and 2-dimensional (2*D*) regular lattices. Here the number of industrial relationships of a company i corresponds to the degree, k_i, of the company. The degree distribution of RN is generally known to be the Poisson distribution, which means that the degree distribution for RN is homogeneous. To generate RNs we use the Erdös–Rényi network model [25]. In contrast to RN, the degree distribution of SFNs is highly heterogeneous which is characterized by a power-law, $P(k) \sim k^{-\gamma}$. In the following simulations we use the Goh et al.'s model to implement SFN for underlying topology [26]. In this SFN model, a weight $w_i = i^{-\alpha}$ is assigned to each node i $(i = 1, 2, \ldots, N)$, where $0 \leq \alpha < 1$. By adding a link between unconnected nodes i and j with probability $w_i w_j / (\sum_{n=1}^{N} w_n)^2$, one can obtain a network whose degree distribution satisfies a power-law $P(k) \sim k^{-\gamma}$. In this SFN model, γ is related to α as $\gamma = (1 + \alpha)/\alpha$. Thus, by adjusting α we obtain a network with any γ (> 2).

4. Market index

The market index is a measure of aggregated market value and is usually defined by the weighted average of market value of equities of listed companies [27]. Thus, the companies which have high price and large number of outstanding shares contribute more to the market index than the others which have low price and small number of outstanding shares. To define the market index in our model, we assume that the companies which have a large number of outstanding shares have more industrial relationships than others which have small market value of equity. It is a natural assumption because the companies which have a large number of outstanding shares are in general large corporations, such as Apple and Microsoft Corp. These large companies generally have more industrial relationships than the small corporations. Based on this assumption, the simplest definition of market index, $I(t)$, at time t can be written as the weighted average of $p(t)$:

$$I(t) = \frac{1}{N}\sum_{i=1}^{N}\frac{k_i p_i(t)}{\sum_{j=1}^{N} k_j}, \tag{1}$$

where k_i is the degree of the company i in IR networks.

5. The distributions of return and avalanche size

The return, $R(t)$, at time t is usually defined as the logarithmic change of index (or price) [2]

$$R(t) = \ln I(t + \Delta t) - \ln I(t). \tag{2}$$

In our simulations we set $\Delta t = 1$. The broad central region of return distribution of a real financial market is known to be well approximated by the stable Lévy distribution:

$$P_L(R) = \frac{1}{\pi}\int_0^\infty e^{-\lambda |q|^\alpha}\cos(qR)dq, \tag{3}$$

or truncated Lévy distribution (TLD):

$$P(R) = cP_L(R)\left[1 - \Theta(R^* - |R|)\right] \tag{4}$$

with $1 < \alpha < 1.5$ [2,3]. Here α and λ are the exponent and scaling factor, respectively. $\Theta(x)$ is the Heaviside theta function satisfying $\Theta(x < 0) = 0$ and $\Theta(x \geq 0) = 1$. c is a normalization constant and R^* is a cutoff. Eq. (3) approaches to the power-law [2]

$$P_L(|R|) \sim |R|^{-(1+\alpha)}, \tag{5}$$

for $|R| \gg 1$ and is known to be stable when $0 < \alpha \leq 2$. For a practical purpose, to analyze the simulation results we use TLD with exponential cutoff as

$$P(R) = cP_L(R)\exp(-|R|/R^*). \tag{6}$$

In Fig. 1 we display the measured $P(R)$ for various IR topologies. The data in Fig. 1(a) show that $P(R)$ for 1D lattice decays exponentially. However, for higher dimensions, a broad region of all measured $P(R)$ is well approximated by TLD with exponential cutoff (see Fig. 1(b)–(d)). Using the least-squares fitting of the data to Eq. (6), we obtain $\alpha = 0.7(3)$ for both a 2D square lattice and RN. The value of α for the 2D lattice and RN is significantly smaller than that for real markets. However, for SFN we obtain $\alpha = 1.1(2)$ which is larger than those for 2D and RN, and closer to the empirical value for real markets [4,28].

The different behavior in $P(R)$ for each underlying IR topology can be understood from the avalanche size distribution, $P(s)$. In general if $\Delta I(t)$ ($\equiv I(t + \Delta t) - I(t)$) is small enough, then $R(t)$ becomes $R(t) \simeq \Delta I(t)/I(t) \sim \Delta I(t)$. From Eq. (1), the change of index is

$$R(t) \sim \Delta I(t) \sim \sum_{i=1}^{N} k_i\left[p_i(t + \Delta t) - p_i(t)\right]. \tag{7}$$

When IR topology is homogeneous, we can apply simple mean-field type argument in which the fluctuation of k_i can be neglected. Thus, by substituting the average degree, $\langle k\rangle$, for k_i, Eq. (7) becomes

$$R(t) \sim \Delta I(t) \sim \langle k\rangle \sum_{j\in\Gamma} a_j s_j, \tag{8}$$

where Γ is a set of updating events during Δt. s_j is the number of companies whose stock prices are updated for the j update event, which corresponds to the size of avalanche. a_j is a constant representing the increase or decrease of price, i.e. if the price increases (decreases), then $a_j = 1$ ($a_j = -1$). In addition, when $P = 1/2$, the distribution of $a_j s_j$ becomes symmetric, and only a few avalanches can contribute to $\Delta I(t)$ due to a random fluctuation. Therefore, the return would be proportional to a randomly selected avalanche size s and $P(R)$ would be proportional to $P(s)$ when the underlying topology is homogeneous. This can be verified from the measurement of $P(s)$. As shown in Fig. 2(a), $P(s)$ for 1D lattice exponentially decays which agrees with the exponentially decaying $P(R)$ in Fig. 1(a). $P(s)$ for 2D lattice and RN can also

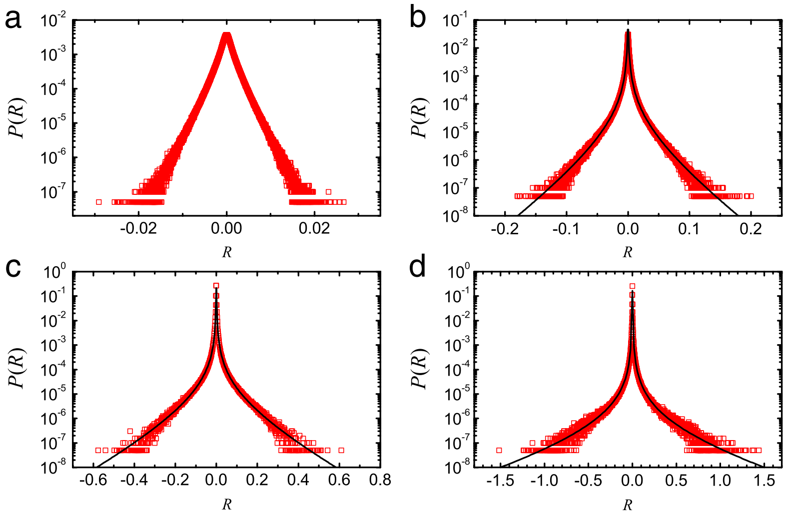

Fig. 1. (Color online) Plot of $P(R)$ for (a) 1D lattice, (b) 2D square lattice, (c) RN, and (d) SFN with $\gamma = 2.5$. The solid lines correspond to Eq. (6) with $\alpha = 0.7$ for both (b) and (c). The solid line in (d) corresponds to $\alpha = 1.1$.

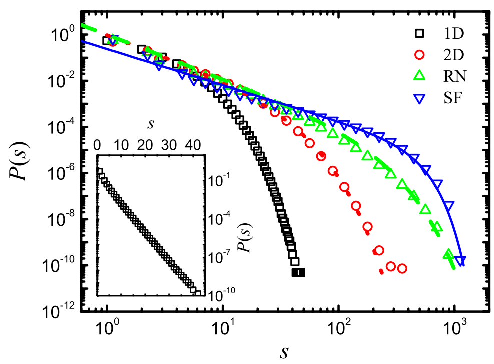

Fig. 2. (Color online) Plot of $P(s)$ for 1D lattice (□), 2D square lattice (◯), RN (△), and SFN with $\gamma = 2.5$ (▽). The dotted line and dashed line represent the relation $P(s) = s^{-1.8}\exp(-s/17)$ and $P(s) = s^{-1.9}\exp(-s/97)$, respectively. The solid line corresponds to the power-law with stretched exponential tail, $P(s) \sim s^{-1.5}\exp(-s^2/s_0^2)$ for SFN. Inset: Plot of $P(s)$ for 1D lattice in semi-log scale.

be well approximated by a power-law with exponential cutoff, $P(s) \sim s^{-\tau}\exp(-s/s_0)$ with $\tau = 1.8(2)$ and $\tau = 1.9(1)$, respectively. These values of τ coincide with the values of $\alpha + 1$ obtained in Fig. 1(b) and (c) within the estimated errors. However, $P(s)$ for SFNs can be well fitted by a power-law with stretched exponential, $P(s) \sim s^{-\tau}\exp(-s^2/s_0^2)$. From the data in Fig. 2, we obtain $\tau = 1.5(1)$ for SFN. The obtained value of τ does not coincide with $\alpha + 1 \simeq 2.1$ for $P(R)$ in Fig. 1(d). This discrepancy would come from the heterogeneous degree distribution which causes nontrivial contribution to $\Delta I(t)$ in Eq. (7).

6. Volatility clustering

The amplitude of the return, measured by the absolute value of return, $|R(t)|$, is generally called as a volatility. The dynamic properties of the volatility are closely related to the amount of information arriving at t and are known to be a measure of the financial risk [2]. Thus, the volatility is very important parameter to model the stochastic process in real financial markets. The volatility clustering, which is one of the main features of the real financial market data, means that the periods of quiescence and abrupt changes tend to cluster together in the time-series [4,5].

In Fig. 3, $R(t)$'s for various IR topologies are displayed. The data in Fig. 3(a) show that the amplitude of $R(t)$ for 1D lattice is quite small, and the period of quiescence is not clearly distinguished from the abrupt changes of index. For 2D lattice, RN and SFN with $\gamma = 2.5$, periods of quiescence and abrupt changes are clustered together (see Fig. 3(b)–(d)). However, the amplitude of $R(t)$ for 2D lattice is still small compared to those for RN and SFN as shown in Fig. 3(b). This indicates that the clustering in $R(t)$ for 2D lattice is weaker than those for RN and SFN.

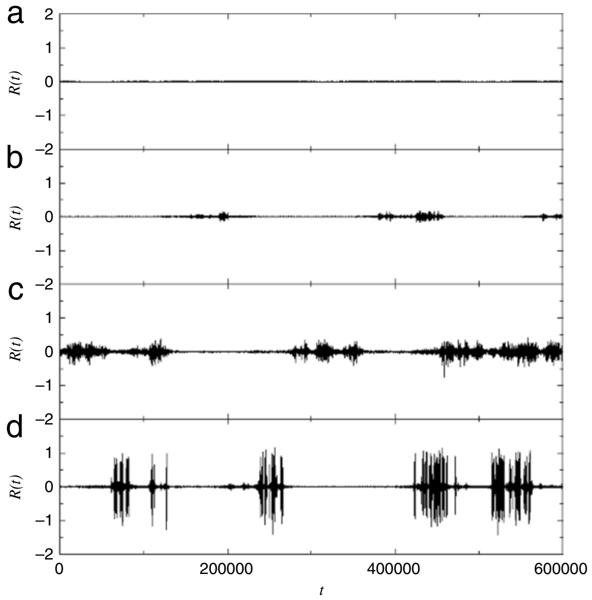

Fig. 3. Plot of $R(t)$ for (a) $1D$ lattice, (b) $2D$ square lattice, (c) RN, and (d) SFN with $\gamma = 2.5$.

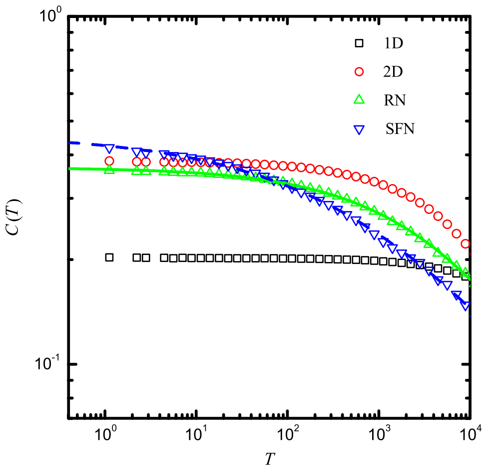

Fig. 4. (Color online) Plot of $C(\tau)$ for $1D$ lattice($\square$), $2D$ square lattice ($\bigcirc$), RN ($\triangle$), and SFN with $\gamma = 2.5$ ($\triangledown$). The lines represent the relation (10). The value of η for each line is $\eta = 0.49$ (green solid line) for RN and $\eta = 0.36$ (blue dashed line) for SFN.

This clustering feature is known to be the origin of the long-term correlation in volatility time series. To measure the time correlation of volatility, we use the autocorrelation function of $|R(t)|$ defined as

$$C(T) = \frac{\langle |R(t)|\,|R(t+T)|\rangle - \langle |R(t)|\rangle^2}{\langle |R(t)|^2\rangle - \langle |R(t)|\rangle^2}. \tag{9}$$

Here $\langle \cdots \rangle$ denotes the time-average. $C(T)$ of volatility for real data in a financial market is known to satisfy a relation

$$C(T) \sim \frac{1}{(T_0 + T^{\eta})}, \tag{10}$$

with $\eta \approx 0.3$ [4]. Here T_0 is a constant. In Fig. 4 we display the measured $C(T)$'s from the simulations. As shown in Fig. 4, $C(T)$ for $1D$ lattice stays at the constant value $C(T) \approx 0.2$ when $T < 10^4$ and is relatively smaller than those for $2D$ lattice and complex networks. This small value of $C(T)$ for $1D$ lattice indicates that the volatility correlation on $1D$ lattice is much closer to that of random fluctuation. $C(T)$ for $2D$ lattice also stays some constant value $C(T) \approx 0.4$ when $T < 500$ then decays almost exponentially, which implies that $\eta \to 0$ for both $1D$ and $2D$ lattices. The large value of $C(T)$ followed by an exponential decay for $2D$ lattice indicates the absence of the long-term correlation. The absence of the long-term correlation

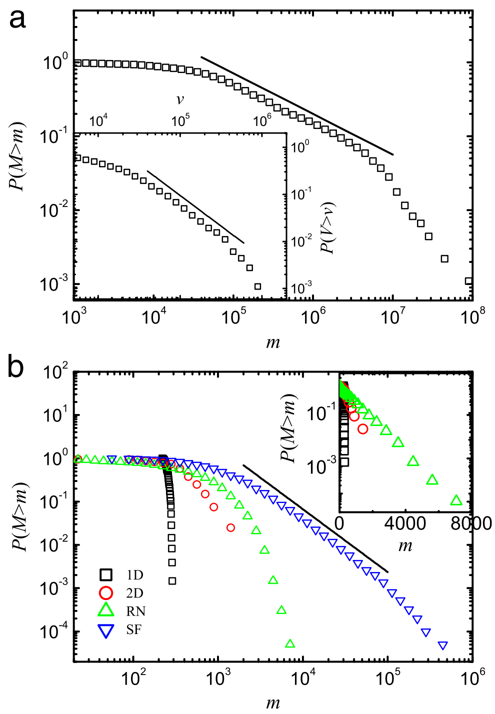

Fig. 5. (Color online) (a) Cumulative distribution of market value of equity for KRX. The solid line represents the power-law $P(m) \sim m^{-\delta}$ with $\delta = 1.55$. Inset: Cumulative distribution for the total number of outstanding shares. The solid line denotes the relation $P(V > v) \sim v^{-1.3}$. (b) Cumulative plot of $P(m)$ for 1D lattice (□), 2D square lattice (◯), RN (△), and SFN with $\gamma = 2.5$ (▽). The solid line corresponds to $\delta \simeq 2.45$. Inset: Cumulative plot of $P(m)$ for 1D lattice, 2D square lattice, and RN in semi-log scale.

for 2D lattice can be explained by the weak volatility clustering as in Fig. 3(b). Therefore, $C(T)$ for both 1D and 2D lattices cannot explain $C(T)$ for real markets. On the other hand, $C(T)$'s for complex networks are well approximated by Eq. (10) as shown in Fig. 4. The best fits of the data to Eq. (10) give $\eta = 0.49(1)$ for RN and $\eta = 0.36(1)$ for SFN, respectively. Indeed, the obtained value of η for SFN is very close to the known value for real market data. Such nontrivial correlations in volatility time series for RN and SFN agree with the data in Fig. 3.

7. Distribution of market value of equity

The market value of equity of company i, m_i, is defined as

$$m_i = v_i p_i, \tag{11}$$

where v_i is the total number of outstanding shares of the company i and p_i is its stock price. Thus, m_i represents the market value or the wealth of the company i. In many economic systems, the wealth distribution satisfies the power-law [29]

$$P(m) \sim m^{-\delta}, \tag{12}$$

which is known as Zipf–Pareto's law. In order to check if the distribution of market value of equity in a stock exchange market also satisfies Eq. (12) or not, we investigate the distribution of market value of equity in Korea Exchange (KRX) for ten years (from 1 July 1997 to 12 February 2008). In Fig. 5(a) we display the cumulative distribution of market value of equity, $P(M > m)$ in KRX. As shown in the figure, $P(M > m)$ satisfies Eq. (12) with $\delta \approx 1.5$ over almost two decades. In the inset of Fig. 5(a) we display the cumulative distribution for the total number of outstanding shares, $P(V > v)$, measured in KRX. The data show that the total number of outstanding share also satisfies a power-law $P(V > v) \sim v^{-1.3}$. This indicates that the distribution of v is not homogeneous and the heterogeneity in $P(v)$ can be one possible origin of Pareto–Zipf's law in $P(m)$.

The contribution of $P(v)$ to $P(m)$ can be easily verified through our model. Since we define the index of stock exchange market as Eq. (1), it is natural to define the market value of equity of each company i of degree k_i as

$$m_i = k_i p_i. \tag{13}$$

Thus $v \sim k$ and the distribution of v becomes heterogeneous for SFN but is homogeneous for 1D lattice, 2D lattice, and RN. In Fig. 5(b) we show the measured $P(M > m)$ in our simulations. The data in the inset of Fig. 5(b) clearly show that $P(M > m)$'s for 1D lattice, 2D lattice, and RN decay exponentially. However, $P(M > m)$ for SFN has a power-law tail over almost three decades with $\delta \approx 2.45$. This indicates that the heterogeneity in $P(v)$ significantly affects $P(m)$ and $P(m)$ only for SFN follows Zipf–Pareto's law (Eq. (12)).

8. Summary

In summary, we propose a stochastic model which mimics the behavior of technical agents in stock exchange markets. In this model the behavior of technical traders causes the avalanches of price changes in the market. When IR topology is homogeneous, the observed behavior of $P(R)$ can be explained by mean-field type arguments based on the measured $P(s)$. However, the obtained value of α for homogeneous IR topologies is far from the empirical value for real markets. On the other hand, when the IR network becomes heterogeneous, $P(R)$ cannot be simply understood from $P(s)$. But the value of α becomes much closer to the empirically known value. From the measurement of the autocorrelation function of the volatility, we find that the value of η for heterogeneous IR network is much closer to that for real markets than η for homogeneous IR topologies. Moreover, we find that $P(v)$ significantly affects $P(m)$ and the heterogeneity of v is the origin of Zipf–Pareto's law in market value of equity distribution. These results clearly indicate that the one possible origin of stylized facts observed in various financial markets is the heterogeneity of the underlying IR topology. In addition, since the existence of cross-correlations between price change and volume change was recently suggested [30,31], it would be an interesting open question that how such cross-correlation would affect the universal behaviors.

Acknowledgments

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