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Interface depinning in a driven growth model with avalanches in quenched random media

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Abstract

A growth model in quenched random media which has the mean-field-like driving force F and local avalanche processes simultaneously and explicitly has been suggested and studied by simulations. It has been found that this model belongs to the same universality class as the directed percolation depinning models. The critical moving regime of the suggested model has been found to be critically the same as a self-organized depinning model. Some discussions on the relation between the self-organized model and DPD models through our model have also been given. © 1999 Elsevier Science B.V. All rights reserved.

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Recently there have been many studies on the depinning of the interfaces in quenched random media by the driving force F [1–8,10–12]. Such interface growths are known to have three regimes. If F is weak (*pinned regime*), then the driven interface is eventually pinned. When F is increased to the critical force F_c which barely overcomes the pinning force, then the interface begins to move. For F just above F_c (*critical moving* (CM) *regime*) the velocity v of the interface follows the power law $v \sim f^{\theta}(f = (F - F_c)/F_c)$. If $F \gg F_c$ (*moving regime*), v increases linearly with F. One of the possible continuum equations [4,5] for such interface growths is the Quenched Kadar–Parisi–Zhang (QKPZ) equation [5]

$$\frac{\partial h(x,t)}{\partial t} = v\nabla^2 h(x,t) + \lambda(\nabla h)^2 + \eta(x,h) + F, \qquad (1)$$

where $h(x,t)$ is the height of the interface and $\eta(x,h)$ is the quenched random force with short-range correlations.

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Among the discrete lattice models which follow the QKPZ equation, there are models which are called the *directed percolation depinning* (DPD) models [6–8]. The width W of the interface in such models satisfies the dynamical scaling law as $W = L^{\alpha} f(t/L^{z})$ [9], where the function $f(x)$ is x^{β} (with $\beta = \alpha/z$) for $x \ll 1$ and is constant for $x \gg 1$. In the CM regime (i.e., $F \sim F_c$) of the DPD model, values of the exponents θ, α and β are known as [6–8,10,11]

$$\theta = 0.59\text{–}0.70, \ \alpha = 0.63\text{–}0.70, \ \beta = 0.63\text{–}0.70. \quad (2)$$

An interesting and variant growth model in quenched random media was recently suggested by Sneppen [12]. Unlike the DPD models, the B model of Ref. [12] (Sneppen B Model) does not have the driving force F for the pinning–depinning transition. Instead the growth is initiated by choosing the site with the global minimum of quenched forces on the interface and the B model has the avalanche process to keep the magnitude of the slopes of the interfaces less than 1 or to keep the restricted solid-on-solid (RSOS) condition [13] globally at any stage of the growth. In 1+1 dimension, the exponent α of the B model is approximately 0.63 which is very close to that of the CM regime of the DPD model. Some of the scaling properties of the B model have been shown to coincide with those of the CM regime of the DPD model [14,15]. In this sense this model is called the depinning model of the self-organized criticality (SOC). Although some researches have been done to understand the physical relation between this model and the DPD models [14–17] by studying several different aspects of the B model, there has been no attempt to understand the model directly from a stochastic model that has both the driving force F and the avalanche processes explicitly. We want to show that the dynamical behavior of the B model is quite the same as that of the CM regime of such a model, i.e., that the B model has a short cut which selects the CM regime of our model. Another merit of the establishment of the present model is that one can measure the dynamical exponent β in two different ways. One way is to use the time scale based on the growth attempts which is the method usually taken in DPD models and the other is to use the time scale based on the actual growth [17]. As we will show by the data in Fig. 3, the discrepancy between the $\beta(\simeq 1)$ of the B Model [12] and $\beta(\simeq 0.69)$ of the DPD model [6–8] can be understood from the results of the two different measurements of β in our model. Another motivation to study our model is that the various exponents such as θ and ϕ which is related to the QKPZ nonlinearity [10,11,18] can be defined and measured in our model, while in Sneppen B model such exponents cannot be defined and measured. Through measurement of θ and ϕ in our model we believe to understand intriguing Sneppen B model more directly.

One of the methods to establish such a model is to use a mean-field-like driving force. It will be shown that the model with the uniform driving force on each site of the interface and local avalanches belongs to the same universality class as DPD models. The details of our model are as what follows. In $1+1$ dimension the model is defined with a column coordinates i, $i = 1, 2, \ldots, L$ and with a height coordinate h. A uniformly distributed uncorrelated random number $\eta_i(h)$ between 0 and 1 is assigned

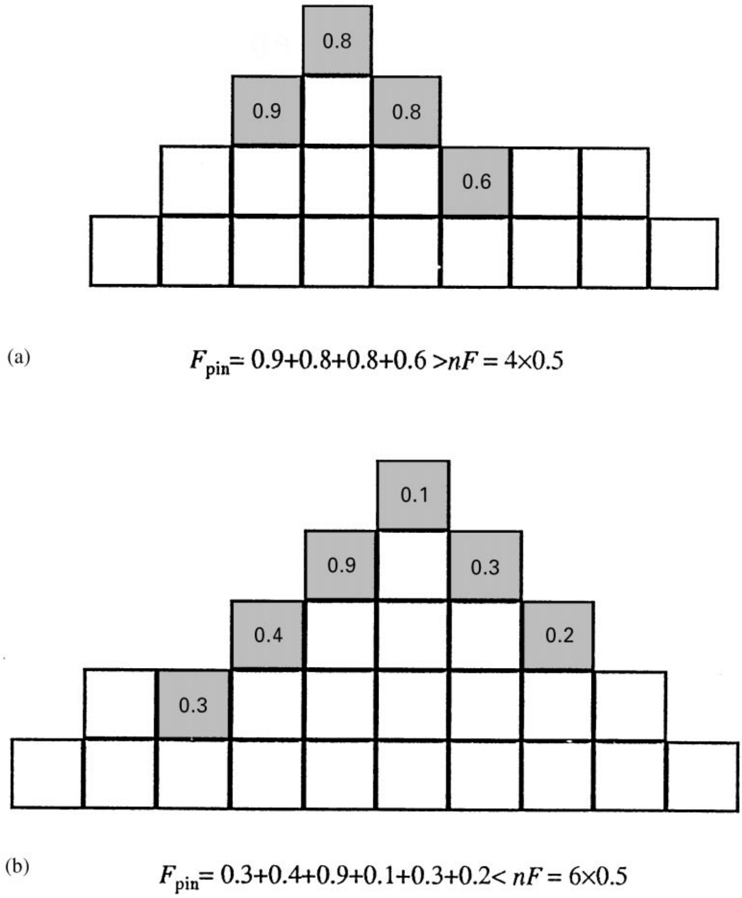

Fig. 1. Two examples for the growth rules (ii)–(v), when $F = 0.5$. The shaded boxes represent the growth zone for each growth attempt. In case (a) the growth zone have 4 columns $n=4$, $F_{\rm pin}=3.1$, and $F_{\rm pin} > nF$. The growth attempt is not allowed. In case (b) $n = 6$, $F_{\rm pin} = 2.2$, and $F_{\rm pin} < nF$. The growth attempt is allowed and the shaded boxes become fresh grown sites.

to each lattice point (i,h). The interface h_i is grown from the initial flat line $h_i = 0$. The interface is updated by the following steps. (i) Select a column at random. (ii) Calculate the force $F_{\rm pin}$ to grow the selected column and to grow the neighboring columns to satisfy the RSOS condition $|h_i - h_{i+1}| \leqslant 1$ on every column i. If this growth zone starts from the jth column to $(j+n-1)$th column, then $F_{\rm pin} = \sum_j^{(j+n-1)} \eta_i(h)$. (iii) Calculate nF, where F is a mean-field-like uniform force and n is the number of columns in the growth zone of the step (ii). (iv) If $F_{\rm pin} \leqslant nF$, then all the growths $h_i + 1$ at the columns in the growth zone are permitted. Otherwise, a new column is selected at random [see Fig. 1 for examples of the processes (ii)–(v)].

Fig. 2 shows the dependence of the growth velocity $v(= {\rm d}\langle h\rangle/{\rm d}t)$ on f (or F) for the substrate of the size $L = 4096$ on the log–log plot. Each data is taken in the

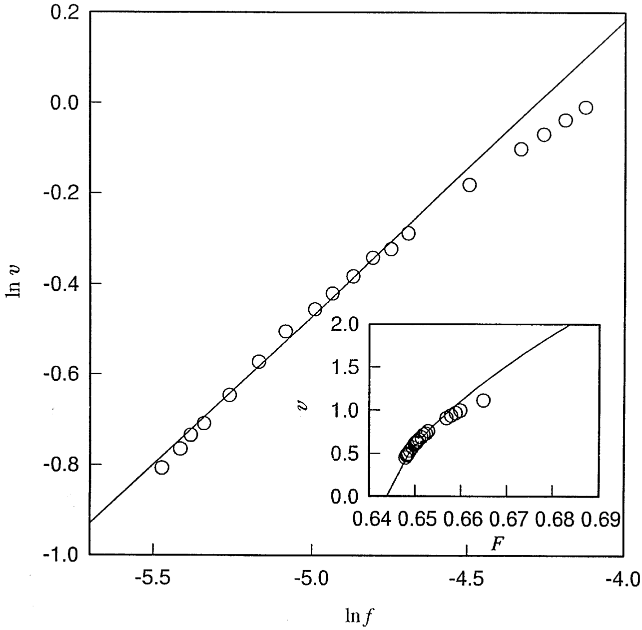

Fig. 2. Plot of $\ln v$ against $\ln f$ for $d = 1 + 1$ in CM regime. Inset shows the plot of v against F.

saturated state ($t \gg L^z$) by averaging over 100 independent runs. By fitting these data to $v \sim (F - F_c)^\theta$, we obtained the critical driving force as $F_c = 0.644 \pm 0.005$ and the velocity exponent as $\theta = 0.63 \pm 0.03$. This result for θ is consistent with that of the DPD models [10,11] (see Eq. (2)).

We measured the early time behaviors of $W(t)$ for $F = 0.65(\simeq F_c)$ using the two different time scales. We used a substrate of size $L = 8192$. The results are shown in Fig. 3. By fitting the data for $W(t)$ based on the time scale of the growth attempts to $W(t) \sim t^\beta$, we obtained $\beta = 0.68 \pm 0.01$. This β-value is nearly the same as that of the CM regime in DPD models. In contrast we obtained $\beta = 1.03 \pm 0.05$ from the data based on the scale of actual growths. The result for the actual growth time in the CM regime is quite close to the β-value of Sneppen-B model [12] and of DPD models for the actual growth time [17]. As explained in the part where the motivation of our study is introduced, the discrepancy between $\beta(\simeq 1)$ of the B Model [12] and $\beta(\simeq 0.69)$ of the DPD model [6–8] can be physically understood from the results in Fig. 3.

To determine the roughness exponent α in the CM regime ($F = 0.65$), we measured the local widths $w(l)$'s for the windows whose sizes are $l = 64, 128, 256, 512$ on the substrate size $L = 8192$ in the saturated state. From the data in Fig. 4 and the relation $w(l) \sim l^\alpha$ [6], we obtained $\alpha = 0.63 \pm 0.01$. This α value is nearly the same as those of the CM regime of the DPD models (Eq. (3)) and of Sneppen B model [12]. This result is an expected one if one believes that the CM regime of our model is critically the same as the B model.

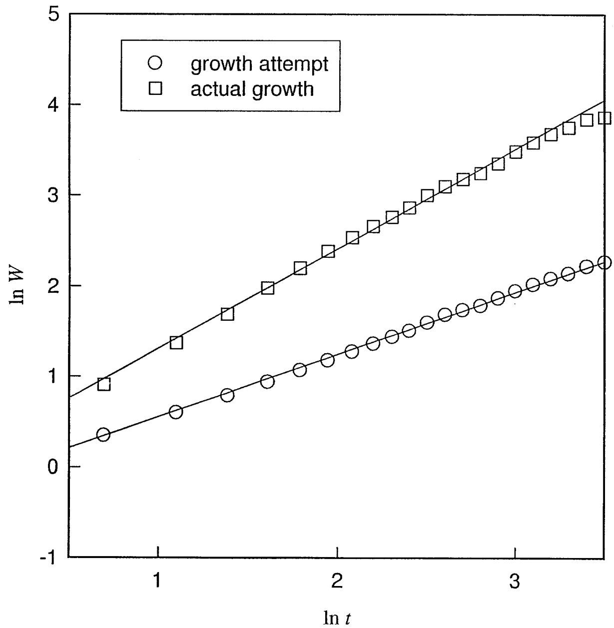

Fig. 3. Plots of $\ln W$ against $\ln t$ using two different time scales.

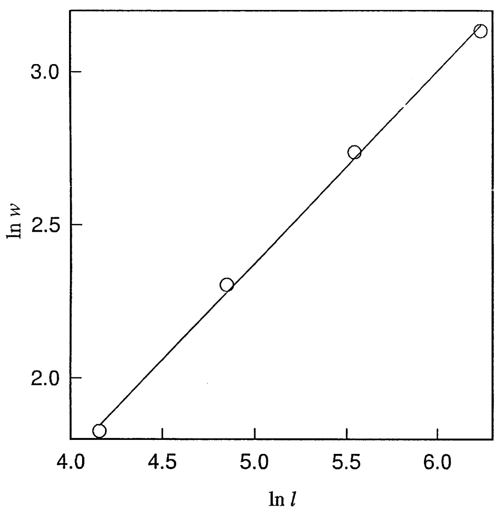

Fig. 4. ws in a saturated state on a square lattice as a function of the local window size $l = 64, 128, 256, 512$.

To understand our model more deeply we measured the tilt-dependent velocity $v(m)$ on the substrate of size $L = 8192$ for several F's in the CM regime. The results are shown in Fig. 5a. $v(m)$ is generally believed to depend on the initial tilt m through the function $v(m) = v(0) + \lambda m^2$ [19], where λ is the coefficient of KPZ nonlinear term

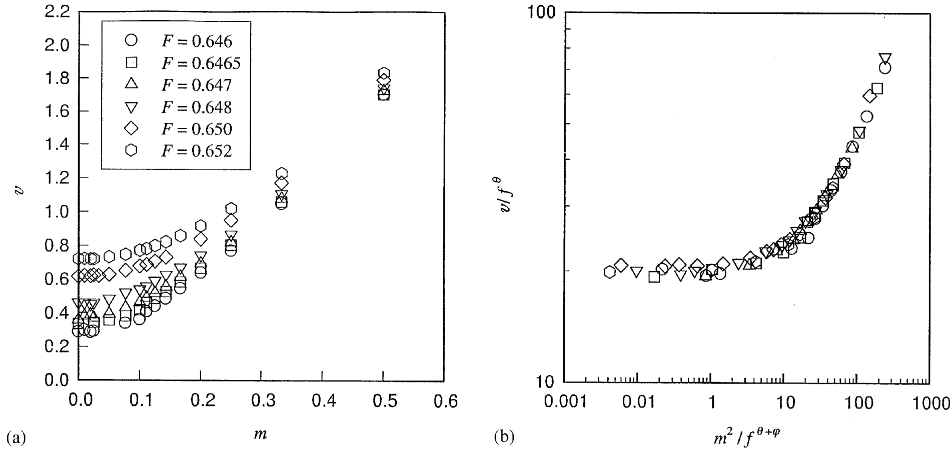

Fig. 5. (a) Dependence of v on the tilt m for the various Fs in CM Regime. m is the slope of the substrate. (b) Data collapse of the velocities using the relation $v \sim f^{\theta} g(m^2/f^{\theta+\phi})$. Here the used values of the exponents are $\theta = 0.69$, $\phi = 0.57$ and $F_c = 0.644$.

$|\nabla h|^2$ in Eq. (1). Since the data in Fig. 5a do depend on m, there exists a non-zero λ near F_c in our model. From this result we can conclude that the CM regime of our model has the KPZ nonlinearity. To study this fact more quantitatively, we analyzed the data for $v(m)$ by using the scaling relation $v(m, f) \sim f^{\theta} g(m^2/f^{\theta+\phi})$ suggested by Amaral et al. [10]. As one can see from Fig. 5b, the data for $v(m)$ is best fitted to the scaling relation when $\theta = 0.69$, $\phi = 0.57$ and $F_c = 0.644$. These values of F_c and θ are consistent with the corresponding values obtained from Fig. 2. The values of the exponents ϕ and θ are nearly the same as those obtained in the DPD models through the measurement of $v(m)$ and the same scaling relation [10,11].

We have also studied the Moving regime (the regime with $F \gg F_c$) of our model and found the moving regime belongs to thermal KPZ universality class as the moving regime of DPD models and as Sneppen B model without any random pinning force η [6–8,11,12,17,20].

From these numerical results we can conclude that the established uniformly driven growth model with avalanches for the RSOS condition in quenched media belongs to the same universality class as DPD models. Especially it has been found that β from the time scale based on the actual growth is nearly the same as that of Sneppen B model, whereas β from the time scale based on the growth attempts is quite close to that of the DPD models. In addition to these exponents we have measured θ and ϕ which cannot be defined in Sneppen B model and the values of θ and ϕ are nearly the same as those of DPD models.

Final discussions are on two points. The first one is on the relation of our model and Sneppen B model. Except for the numerical results which have supported that the saturated state of CM regime of our model is critically the same as that of Sneppen B model, we have more direct argument for this. The argument is related to the value of F_c. $F_c \simeq 0.65$ in our model can be explained from the property of the saturated state of Sneppen B model. If the saturated state of our model were just the same as that of the Sneppen B model, then F_c should be estimated from the growth of the growth zone which consists of the site with the largest $\eta_{\min}$ and its neighbors for the avalanches. Here $\eta_{\min}$ is the minimal η of the quenched noises along a certain interface. According to Ref. [16] the largest $\eta_{\min}$ is 0.461. The characteristic avalanche size s in the Sneppen B model is known to be about four [12] and η of these avalanche sites are uniformly distributed in the interval $[0.461, 1]$ [14]. The average pinning force $\langle\eta\rangle$ of each avalanche site is about 0.731. Thus from $s \times F_c = 0.461 + (s-1) \times 0.731$ with $s = 3 \sim 4$, the estimated F_c around 0.67, which is close to the value of F_c obtained from Fig. 2 and Fig. 5. This coincidence also gives another proof that the saturated state of CM regime of our model is critically the same as that of Sneppen B model.

The second discussion is on the parallel and perpendicular correlation lengths $\xi_{\parallel}, \xi_{\perp}$ [21] of the interface. In DPD models the interface is believed to be pinned by the directed percolation path [6,8] and the exponents $v_{\parallel}$ and $v_{\perp}$ [21] of DPD models are known to be $v_{\parallel} \approx 1.73$ and $v_{\perp} \approx 1.10$ [8,10,21]. The exponents $v_{\parallel}$ and $v_{\perp}$ of our model can also be estimated from the two relations $\theta + \phi = 2v_{\parallel}(1-\alpha)$ [18] and $\alpha = v_{\perp}/v_{\parallel}$ [11,12,16]. From numerical results for θ, ϕ and α of our model the estimated values for $v_{\parallel}$ and $v_{\perp}$ of our model are $v_{\parallel} \approx 1.73$ and $v_{\perp} \approx 1.10$. These values are nearly the same as those of DPD models [21,22]. The interface of our model thus should be pinned by nearly the same path as the directed percolation path. This fact also supports that the CM regime of our model belongs to the same universality class as DPD models. Two final discussions should explain more physically why the numerical results of our model are consistent with Sneppen B model and DPD models and thus explain the relationship between Sneppen B model and DPD models more clearly.

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