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Agent-Based Generalized Spin Model for Financial Markets on Two-Dimensional Lattices

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We study a microscopic model for price formation in financial markets on a two-dimensional lattice, motivated by the dynamics of agents. The model consists of interacting agents (spins) with local and global couplings. The local interaction denotes the tendency of agents to make the same decision as their interacting partners. On the other hand, the global coupling to the self-generating field represents the process which maximizes the profit of each agent. In order to incorporate more realistic situations, we also introduce an external field which changes in time. This random field represents any internal or external interference in the dynamics of the market. For the proper choice of model parameters, the competition between the interactions causes an intermittency dynamics and we find that the distribution of logarithmic return of price follows a power law.

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I. INTRODUCTION

Power law or fat tails observed in various distributions [1–3] in many economic systems have attracted physicists' attention because of their relevance to critical phenomena in statistical physics. Especially, empirical studies on logarithmic price changes (returns) in real markets have found the intermittent occurrence of large bursts resulting in the power law tails in their distributions. Various models have been introduced to understand the origin of the observed properties in real financial markets [3–9]. Among these models, Ising-like spin systems have been studied by some researchers, due to their simplicity [6–8]. For example, Chowdhury and Stauffer [6] introduced a super-spin and time-dependent individual bias of each agent to show that the return distribution satisfies a power law scaling.

Recently, a particularly realistic model was studied by Lux and Marchesi [10]. In their model, the pool of agents is divided into two groups: "fundamentalist", who exactly knows the excess demand (difference between the demand and supply), and the "trend follower" or "noise trader", who follows the decision of their interacting neighbors [10]. The resulting model succeeded in reproducing several non-trivial properties in a real market. However, the only drawback of the Lux-Marchesi model is a high complexity, with more than ten free parameters. There is thus a requirement for more simplified models to systematically understand the effects of the fundamentalist and the trend follower on the market dynamics [7,8,11]. For example, Sznajd-Weron and Weron studied the effects of a single fundamentalist by using Ising-like ferromagnetic interactions on two sublattices [8] and showed that the resulting return distribution satisfied the power law scaling. Another interesting spin model which incorporates the competition between the fundamentalist and trend follower was suggested by Bornholdt [7]. In Bornholdt's model, each agent is assumed to have features of both the fundamentalist and the trend follower. The ferromagnetic interaction with nearest neighbors is used to stand for the characteristics of the trend followers. At the same time, each spin (or agent) interacts with the global magnetization. The global interaction represents the tendency of the fundamentalist by encouraging a spin flip when the global magnetization becomes large. This interaction depends only on the magnitude of the magnetization, but not the current state of a spin. Thus, the global interaction in Bornholdt's model implicitly contains the effective time-dependent bias of each spin, which is similar to that of Chowdhury and Stauffer's model. As a result of competition between two interactions, the frustration caused by metastable dynamics with intermittency is observed and the resulting return distribution exhibits power law scaling.

In this paper, we study the origin of the power law distribution of the return based on a microscopic dynamics between many interacting agents on square lattices as a starting point to understand the effects of trend followers and fundamentalists. For more systematic analysis, we modify the global interaction of Bornholdt's model by assuming that each spin changes its state if the direction of the global magnetization of the system is parallel to

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the spin orientation. Thus, our model incorporates the competition between the characteristics of the trend follower and the fundamentalist, and time-dependent global bias, which represents any internal and external interference in the market dynamics. By numerical simulations, we find that the competition between trend follower and fundamentalist causes formation of domains. When this domain structure is completely destroyed by the external field and then restored, we find a large burst in the return and the resulting return distribution follows the power law or fat-tailed distribution. In real markets, all agents do not have the same number of interacting partners. The effects of the heterogeneity of the underlying topology characterizing the structure of interaction between agents will be discussed elsewhere [12].

II. MODEL

In our model, we consider $N = L \times L$ agents placed on a square lattice with the linear dimension L. The state of each agent i is described by a two-state spin variable $s_i(t) \in \{-1, +1\}$ at time t. Each state represents buying $(+1)$ or selling (-1). The state of each agent i at time $t+1$, $s_i(t+1)$, depends on the Hamiltonian:

$$H(t) = -\sum_{i,j=1}^{N} J_{ij} s_i(t) s_j(t) + \sum_{i=1}^{N} s_i(t) \{\alpha M(t) - f(t)\}, \quad (1)$$

and is updated by the heat-bath dynamics for simplicity [13]. Here, $J_{ij} = J$ (> 0) if i and j are nearest neighbors; otherwise, $J_{ij} = 0$ (ferromagnetic interaction). This ferromagnetic interaction represents the tendency that each agent will follow the decision of his cooperating agents (characteristics of the trend follower). In the second term, $M(t) = (1/N)\sum_j^N s_j(t)$ corresponds to the excess demand. For $\alpha > 0$, the interaction becomes antiferromagnetic and stands for the tendency of the fundamentalist who exactly knows the excess demand. The antiferromagnetic interaction reflects the fact that if the demand (supply) exceeds the supply (demand), then each agent wants to place a selling (buying) order to maximize his benefit. $f(t)$ in the third term denotes a time-dependent external field which incorporates all internal and external interference in the market dynamics. In general, the magnitude of this interference changes in time and large interferences such as oil shock and subprime mortgage crisis do not have the same occurrence probability as negligible daily reported rumors. Thus, at each time step we choose $|f(t)|$ from the power law distribution:

$$P_f(|f|) \sim \frac{1}{|f|^{\sigma}}, \quad (2)$$

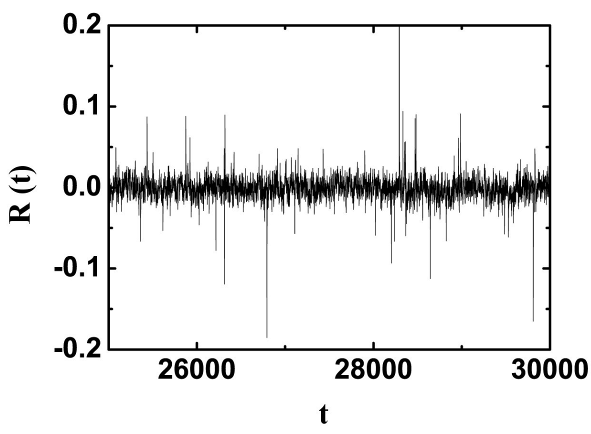

Fig. 1. Evolution of return on a 2-d square lattice. Intermittent large bursts are shown.

where σ determines the heterogeneity of the distribution. The sign of $f(t)$ is chosen at random. The unit time step is defined by the usual Monte Carlo time step.

III. RESULTS

For simplicity, we assume that the evolution of price, $p(t)$, follows [14]

$$\frac{dp(t)}{dt} = cM(t)p(t), \quad (3)$$

where c is a scaling factor for price change. In a discrete time step, $p(t+1) = p(t)\exp(cM(t))$. Thus, the logarithmic return becomes

$$R(t) = \ln p(t) - \ln p(t-1) = cM(t-1). \quad (4)$$

In the following simulations, we use square lattices with $L = 300$. For simplicity, let the Boltzmann constant k_B and J be unity. Figure 1 shows the time evolution of $R(t)$ when $\sigma = 2.5$, $\alpha = 4$ and $T = 2.0$. When $\alpha = 4$, the antiferromagnetic interaction becomes comparable with the ferromagnetic interaction. With these values of parameters, we find the intermittent occurrence of a large burst in $R(t)$, which is very similar to that of real market indices [15,16].

In Figure 2 we display $P(|R|)$ and $P(R)$. In general, $P(R)$ of the real market has been approximated by a Lévy stable distribution [3,17]:

$$P(R) \equiv \frac{1}{\pi}\int_0^{\infty} \exp\left(-\gamma |q|^{\mu}\right)\cos(qR)dq. \quad (5)$$

Here, μ and γ are the index and scaling factor, respectively. For large $|R|$, Eq. (5) can be written as a power law:

$$P(|R|) \sim |R|^{-(1+\mu)}. \quad (6)$$

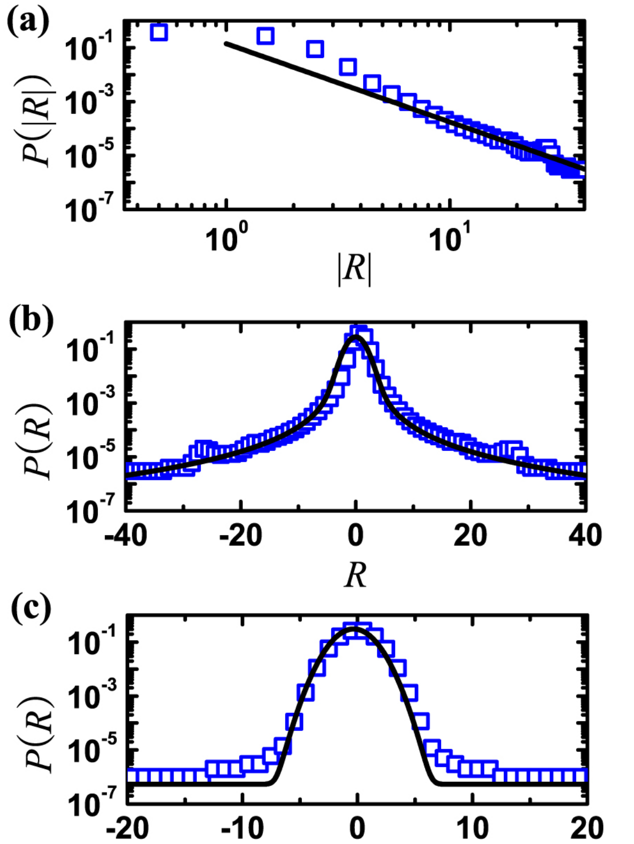

Fig. 2. (a) Plot of $P(|R|)$ against $|R|$ for $\alpha = 4.0$, $\sigma = 2.5$, and $T = 0.5$. The solid line represents the relation $P(|R|) \sim |R|^{-1.9}$. (b) $P(R)$ for the same parameters as in (a). The solid line in (b) displays the Lévy distribution with the μ obtained from (a). (c) Plot of $P(R)$ *vs.* R for $\alpha = 4.0$, $\sigma = 3.5$ and $T = 2.0$. The solid line represents the Gaussian distribution.

From the best fit of Eq. (6) to the data in Figure 2(a), we find $\mu = 1.9 \pm 0.1$ for $|R| > 50$ when $T = 0.5$, $\alpha = 4.0$ and $\sigma = 2.5$. In Figure 2(b), we plot the $P(R)$ obtained and Eq. (5) with $\mu = 1.9$. The result shows that $P(R)$ obtained from the simulations agrees very well with Eq. (5). As σ increases, $P_f(|f|)$ becomes relatively homogeneous. Thus, the price fluctuations are not correlated and $P(R)$ becomes Gaussian (for example, see Figure 2(c)). Other results for various values of parameters are listed in Table 1. Note that for small α $(= 1)$ we find that $P(R)$ does not follow the power law distribution when T is less than the critical temperature of the Ising model, T_c $(= 2.269185\cdots)$. Since the coordination number of a two-dimensional square lattice is four, if $\alpha < 4$, then the ferromagnetic interaction always dominates when $T \ll T_c$. Hence, the system is always in the ordered state and we find that $P(R)$ does not exhibit the power law scaling when $T < T_c$ and $\alpha < 4$ (see Table 1).

In order to understand the detailed dynamics which cause the fat-tailed distribution shown in Figure 2, we take a snapshot of the spin configuration at various temperatures without the external field $f(t)$. As shown in Figure 3(a), when $T = 0$ and $\alpha = 4$ there exist two explicit domains, due to the competition between ferromagnetic and antiferromagnetic interactions. The resulting magnetization becomes 0. If we apply the external field f, then the domain structure can be disturbed and $M \neq 0$. For sufficiently large f, $|M|$ becomes close to 1. Since $P_f(|f|)$ is given by Eq. (2), the probability of having large $|f|$ is very small compared with that of small $|f|$. Therefore, $R(t)$ mostly fluctuates around $R(t) = 0$ with intermittent bursts. These occasional high peaks cause the power law tail of $P(R)$. If we increase T slightly, each domain smears into the other domain, due to the thermal fluctuations. However, as long as T is less than T_c and $f = 0$, we still find the domain structure caused by the competition between ferromagnetic and antiferromagnetic interactions (see Figure 3(b)). Therefore, if $T < T_c$ and $\alpha \simeq 4$, then the power law tail of $P(R)$ is expected when the external field disturbs the domain structure or $f \neq 0$.

This result provides a clue to understanding the market dynamics. In our model, one necessary condition for the power law tail of $P(R)$ is a balance between the sellers and buyers. This balance can be achieved by competition between the fundamentalist's feature (global antiferromagnetic interaction) and the characteristics of the trend follower (ferromagnetic interaction). If large time-dependent market interference is applied to this balanced state, then most of the buyers (sellers) are abruptly changed to sellers (buyers). The sudden changes between buyers and sellers cause intermittent occurrence of large

Table 1. Obtained values of μ for various values of α, σ and T $(< T_c)$. $\times$'s in the table indicate that the distribution follows neither the power law (Lévy distribution) nor a Gaussian distribution.

		$T = 0.5$	$T = 2.0$
	σ	μ	μ
$\alpha = 1$	2.5	$\times$	$\times$
	3.0	$\times$	$\times$
	3.5	$\times$	Gaussian
$\alpha = 4$	2.5	1.9 ± 0.1	1.8 ± 0.1
	3.0	3.4 ± 0.1	3.0 ± 0.1
	3.5	Gaussian	Gaussian

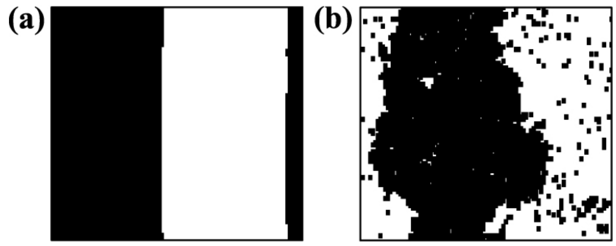

Fig. 3. Snapshot of the spin configurations with $\alpha = 4$ and $f = 0$. (a) $T = 0$, (b) $T = 2$, on a 100×100 square lattice.

bursts, as shown in Figure 1, depending on the nature of the market interference distribution. Therefore, based on the simulations of our model, we expect that the market will always tend to be balanced by the buyers and sellers. Furthermore, the market needs the interference, which can be characterized by a highly heterogeneous distribution such as Eq. (2), to show the power law return distribution.

IV. SUMMARY

We study a two-dimensional generalized spin model on a two-dimensional lattice for price changes in a financial market motivated by the characteristics of agents. The model assumes that each agent has tendencies of both trend follower and fundamentalist. Each tendency is represented by ferromagnetic interaction with nearest neighbors and antiferromagnetic interaction with a global self-generated field. The antiferromagnetic interaction assumes that each agent is smart enough to make his decision to be a minority in the market dynamics. From the numerical simulations we find that the competition between these two interactions causes a domain structure on the two-dimensional lattice. If we apply a time-dependent external field whose magnitude follows a power law distribution, then we find a power law tail of $P(R)$. The emergence of the domain structure indicates that the financial market is essentially balanced by the buyers and sellers, due to the competition between the characteristics of fundamentalist and trend follower. If occasional large interferences in market dynamics are applied to this balanced state, then the power law or fat-tailed distribution of return can be obtained. Based on our results and other empirical studies on real markets [1–3], we expect that most of the real markets will always be in a balanced state through the buyers and sellers. Large market interference which can be characterized by a highly uneven distribution such as a power law distribution is needed for a power law or fat-tailed return distribution.

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REFERENCES

[1] B. B. Mandelbrot, *Fractals and Scaling in Finance* (Springer, New York, 1997).
[2] K. E. Lee and J. W. Lee, J. Korean Phys. Soc. **50**, 178 (2007).
[3] R. N. Mantegna and H. E. Stanley, *An Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, New York, 2000).
[4] R. Cont and J. P. Bouchaud, Macroeconomic Dyn. **4**, 170 (2000).
[5] V. M. Eguíluz and M. G. Zimmermann, Phys. Rev. Lett. **85**, 5659 (2000).
[6] D. Chowdhury and D. Stauffer, Eur. Phys. J. B **8**, 477 (1999).
[7] S. Bornholdt, Int. J. Mod. Phys. C **12**, 667 (2001).
[8] K. Sznajd-Weron and R. Weron, Int. J. Mod. Phys. C **13**, 115 (2002).
[9] K. Kim, S.-M. Yoon, S. Y. Kim, D.-I. Lee and E. Scalas, J. Korean Phys. Soc. **50**, 182 (2007).
[10] T. Lux and M. Marchesi, Nature **397**, 498 (1999).
[11] T. Kaizoji, S. Bornholdt and Y. Fujiwara, Physica A **316**, 441 (2002).
[12] S.-H. Yook, H.-J. Kim and Y. Kim, unpublished.
[13] D. P. Landau and K. Binder, *A Guide to Monte Carlo Simulations in Statistical Physics* (Cambridge University Press, Cambridge, 2000).
[14] M. Bartolozzi and A. W. Thomas, Phys. Rev. E **69**, 046112 (2004).
[15] H.-H. Jo, S. Y. Kim and K. Kim, J. Korean Phys. Soc. **49**, 719 (2006).
[16] P. Gopikrishnan, V. Plerou, L. A. N. Amaral, M. Meyer and H. E. Stanley, Phys. Rev. E **60**, 5305 (1999).
[17] R. N. Mantegna and H. E. Stanley, Nature **376**, 46 (1995).