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Size of Local Avalanches and Self-Organized Interface Growth

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Modified Sneppen models in which local avalanches of the size $s \leq s_c$ are only allowed are suggested and studied by simulations. The models with fixed cutoff size s_c's belong to the same universality class as the Sneppen A model, while the models with s_c proportional to substrate size L ($s_c \propto L$) are critically the same as the Sneppen B model. The results for the modified models are also considered from the measurements of the probability distribution of s in the saturation regime of the B model.

Since the Kadar-Parisi-Zhang (KPZ) equation [1] was suggested for kinetic interface roughening phenomena, there have been many theoretical and experimental studies on the interface formation in thermal white noises [2,3]. However, some experiments have indicated that there exist interfaces which do not belong to the KPZ universality class [4–9]. For the theoretical interpretation for such interfaces, several growth models with quenched randomness [8–14] have been suggested. Recently, Sneppen [14,15] has suggested two interesting models with quenched impurities. In the Sneppen A model, the growth is permitted at the site with the smallest quenched force $\eta(x,h)$ among the sites which keep the restricted solid-on-solid (RSOS) condition [16]. In the Sneppen B model, the growth is initiated by choosing the site with a global minimum of quenched forces on the interface, and the B model has a local avalanche process to keep RSOS condition [16] globally at any stage of the growth. The roughness exponent α of the B model is nearly equal to that of the critical regime of the so-called **directed percolation depinning (DPD)** models, and this model is believed to self-tune the interface such that it is very similar to that of the DPD model with the critical applied force [8–10]. In these senses the B model is called the self-organized depinning (SOD) model. Many studies have been done to understand the physical origin of the self-organized behavior in the B model [15,17–21]. Especially, several authors have studied the spatial and temporal correlations in the B model [15,18,19]. These studies have shown that there exists nontrivial behavior, colored activities and multi-affinity in the B model, and to some extent, they have succeeded in explaining the self-organized criticality (SOC) of the B model physically. More recently, a model which crosses over from the B model to a RSOS model has been suggested and studied [22].

In this paper, we want to study the Sneppen models from a different point of view. The fundamental growth process in the A model is essentially local. In contrast, the process in the B model has a nonlocal nature or a global property. Although the average size of the characteristic avalanche length for the RSOS condition was shown to be nearly equal to 4 [15] it cannot absolutely be concluded that no anomalous distribution exists for large s. It is also physically important to check that the local avalanches of large s are crucial to the SOC behavior of the B model. One way to study the importance of the growth processes with avalanches of large s is to introduce a preassigned cutoff size s_c for the avalanches. If the growth processes with local avalanches of $s \leq s_c$ are allowed and if those with local avalanches of $s > s_c$ are rejected, then the critical property of the modified models with the cutoff size s_c can show a complex behavior. If s_c is small, then the modified model is expected to belong to the same universality class as the A model. If s_c is comparable to the substrate size L, then the modified model definitely belongs to the same universality class as the B model. The purpose of this paper is, therefore, to study the modified models by controlling the cutoff size s_c and to understand the Sneppen models through these modified models.

The details of the modified Sneppen models are as follows; We have an interface $h(x,t)$ on a one-dimensional discrete chain $x = 0, 1, 2, \cdots, L$, and a random number $\eta(x, h+1)\,(\in [0,1])$ is assigned to each growth site

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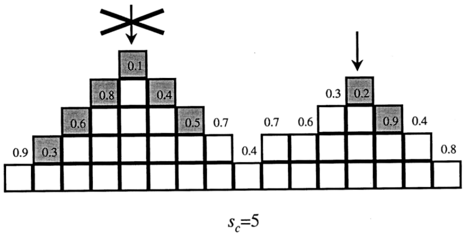

Fig. 1. Schematic illustration of the modified Sneppen model with $s_c = 5$. Shaded sites denote the local avalanches for the given growth processes. The growth process involving the first chosen site ($\eta = 0.1$) is rejected because $s\,(= 6) > s_c$. The process for the next chosen site ($\eta = 0.2$) is allowed because $s(= 2) \leq s_c$.

$(x, h + 1)$. The periodic boundary condition is always imposed. $h(x, t)$ is updated by the following steps: i) Find a column which has the smallest η. ii) The growth $h \to h+1$ is permitted at the found site. iii) Then, the neighboring sites are adjusted ($h \to h+1$) until the magnitudes of the slopes along the interface are less than or equal to 1. iv) Count the number, s, of the adjusted sites in steps ii) and iii); *i.e.*, count the size of local avalanche. v) If s is less than or equal to the preassigned cutoff size s_c (*i.e.*, if $s \leq s_c$), then the growth steps ii) and iii) are permitted and new random numbers are assigned to the new growth sites. Otherwise, the growth steps ii) and iii) are rejected, the site with the next smallest η is found, and the growth steps ii), iii), and iv) are repeated until the condition $s \leq s_c$ is satisfied. The schematic examples for the steps i)$-$v) are given in Fig. 1. If $s_c = 1$, the modified model is exactly the same as the Sneppen

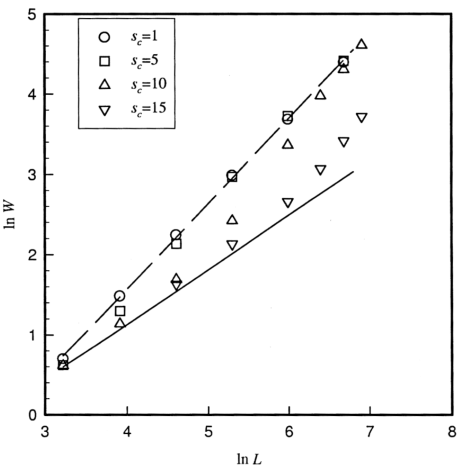

Fig. 2. Plot of $\ln W(L, \infty)$ against $\ln L$ for the models with various fixed s_c's. The solid line corresponds to $W \simeq L^{0.63}$, and the dashed line corresponds to $W \simeq L^{1.0}$.

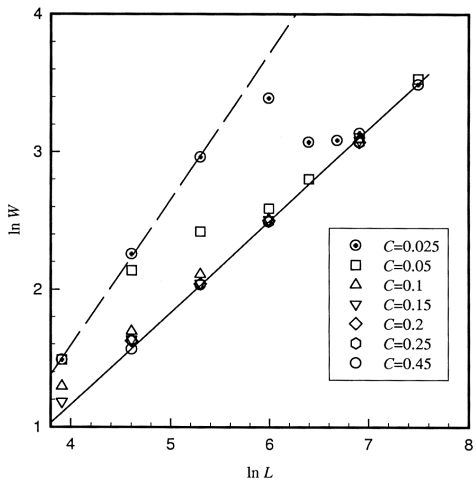

Fig. 3. Plot of $\ln W(L, \infty)$ against $\ln L$ for the models with $s_c = CL$.

A model. If $s_c = L$, the modified model is reduced to the Sneppen B model. If $1 < s_c < L$, then we can expect some nontrivial behavior, and we want to study this nontrivial behavior by the simulations.

From the simulations for the modified models with fixed s_c's, we first obtained the saturated width $W(L, \infty)$ or $W(L, t)$ when $t \gg L^z$. The substrate sizes used are $L = 25$, 50, 100, 200, 400, 600, 800, and 1000. $W(L, \infty)$'s for the models with $s_c = 1$, 5, 10, and 15 are displayed in Fig. 2. $W(L, \infty)$'s in Fig. 2 were obtained by averaging over 50 independent runs. The roughness exponent α of the Sneppen A model is nearly equal to 1.0 [14,15]. The dashed line in Fig. 2 satisfies the relation $W(L, \infty) = L^{1.0}$ and represents the A model. The exponent α of the Sneppen B model is about 0.63 [14]. The solid line in Fig. 2 satisfies the relation $W(L, \infty) = L^{0.63}$ and represents the B model. W's for the models with small s_c's or $s_c \leq 5$ follow the dashed line quite well, and this result coincides with the expectation that the modified models with the small s_c's belong to the same universality class as the Sneppen A model. However, W's for the models with the large s_c's ($s_c \geq 10$) show rather unexpected behavior. On the small substrates ($L \leq 200$), W's for the models with $s_c = 10, 15$ seem to follow the solid line. However, W's deviate from the solid line and approach the dashed line as L increases in the range $L > 200$. From this trend, we can infer the following fact. However large s_c may be, the models with the fixed s_c's on the substrates of very large L belong to the same universality class as the A model. Since the average characteristic length of local avalanches is about 4 [15], this result is rather surprising. This result also suggests that there must be some nontrivial effects of the local avalanches with large s in the Sneppen B model. If the distribution of s in the B model is trivially small for large s, as with the exponential distribution, then we

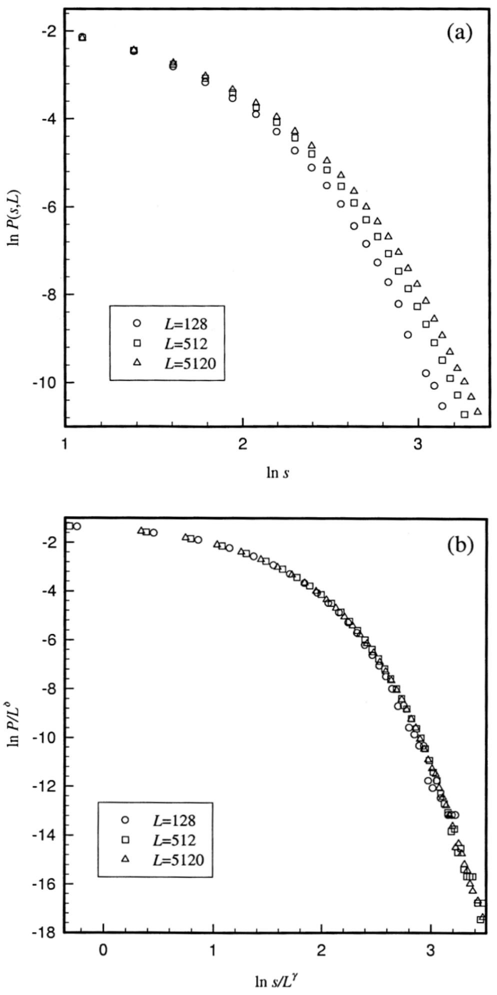

Fig. 4. (a) Plot of the probability distribution $P(s, L)$ of the B model in the saturation regime. (b) The fit of the data in (a) to the scaling of $P(s, L) = L^{\delta} f(s/L^{\gamma})$. The best scaling collapse was obtained when $\gamma = 0.05$ and $\delta = -0.05$.

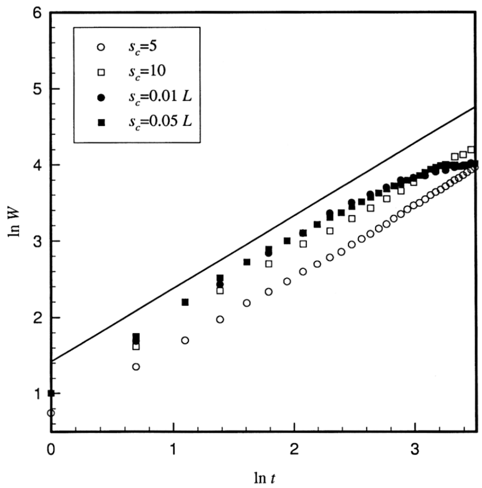

Fig. 5. Plot of $\ln W(t)$ against $\ln t$ for the various modified models. The solid line corresponds to $W \simeq t^{0.95}$.

cannot expect W's as in Fig. 2.

To understand such unexpected results for the modified models with the fixed s_c, we also investigated the modified models with s_c proportional to the substrate size L, *i.e.*, $s_c = CL$. The periodic condition imposes the constraint $0 < C < 1$ on C. Similar simulations were done for these latter modified models as those done for the former models with fixed s_c's, and the results for the saturated widths $W(L, \infty)$'s are displayed in Fig. 3. The solid line in Fig. 3 corresponds to the B model, as in Fig. 2, and the dashed line in Fig. 3 corresponds to the A model, as in Fig. 2. For the large C's ($C \geq 0.15$), W's follow the solid line quite well. In contrast, W's for the small C's ($C < 0.15$) show a rather complex behavior. These W's seem to follow the dashed line for the small L's, but as L increases, W's deviate from the dashed line and approach the solid line. For more detailed explanations, let's scrutinize the data for the model with $C = 0.025$. For $L < 200$, W's follow the dashed line, but for $L > 200$, W's start to deviate from the dashed line and approaches to the solid line. For $L \geq 800$, W's with $C = 0.025$ follow the solid line quite well. From the results in Fig. 3, we can conclude the following fact. However small C is, the modified models with $s_c = CL$ belong to the same universality class as the Sneppen B model. To understand this result more deeply, we measured the probability distribution $P(s, L)$ as a function of s and L in the saturation regime ($t \gg L^z$) of the B model. We measured the $P(s, L)$'s for $L = 128, 512$, and 5120. The data for the $P(s, L)$'s were obtained by averaging over the 10 million particle drops for each substrate, and the results are shown in Fig. 4(a). We tried to fit the data in Fig. 4(a) to the scaling form $P(s, L) = L^{\delta} f(s/L^{\gamma})$. The best scaling collapse of the data was obtained for $\gamma = 0.05$ and $\delta = -0.05$, as can be seen from Fig. 4(b). Since $0 < \gamma < 1$ in the best scaling fit, the results for the modified models can be understood from $P(s, L)$. Since γ is very small compared to 1, we expect that $P(s, L)$ has a very subtle dependence on L. Therefore, deeper and wiser studies will be needed to understand the relation between the avalanches of large size s and the SOC behavior of the B model completely.

Final discussions are on the growth exponent β for the modified models. For this, we measured $W(t)$ when $t < L^z$. The substrate size used for the measurement was $L = 4096$. In the modified models, including the A model, the growth may occur or may not occur for a given growth attempt, while the growth must occur in the Sneppen B model. We, thus, measured the early time $W(t)$ using two different time scales [23]. In Fig. 5, we show $W(t)$'s measured by use of the time scale based on the actual growth. The data in Fig. 5 were obtained by averaging over 50 independent trials. The solid line in Fig. 5 denotes the line for $\beta = 0.95$. From the data for $\ln t < 2$ in Fig. 5 and the relation $W(t) \simeq t^{\beta}$, we obtained $\beta \simeq 0.95$ for all the models with $s_c = 5, 10, 0.01L$, and $0.05L$. We also evaluated the β's for the various modified models mentioned in Figs. 2 and 3 and found that $0.90 < \beta < 0.96$ when the time scale of the actual growth is used. Since the exponent β for both the A and the B models [14] is close to 0.95, these results for the exponent β from the actual growth time are expected results. We also measured the β's by use of a time scale based on the growth attempt. Here, we found some anomalous behavior. For the models with small fixed s_c's ($s_c \leq 5$), the early time $W(t)$'s did not follow the power law $W(t) \simeq t^{\beta}$. However, for the models with $s_c = CL$, $W(t)$'s satisfied the relation $W(t) \simeq t^{\beta}$ relatively well, and the estimated β was about 0.82. Even though this β-value is close to that of some models in quenched random media [12], the physical meaning of this result is still far from being understood.

In this work, we have introduced and discussed the modified Sneppen models with local avalanches of finite sizes. The surface roughenings of the modified models with finite s_c's are critically the same as that of the A model. In contrast, the modified models with $s_c = CL$ belong to the same universality class as the B model. This result suggests that large-sized local avalanches must have nontrivial effects on the SOC behavior of the B model.

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