Fourier Seriese

A Very Basic Thing

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November 25, 2015

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Fourier Seriese

- A (periodic) function $f(t)$ can be expressed as a linear combination of sinusoidal function

Fourier Seires

$$f(t) = \sum_{k=0}^{\infty} (a_k \cos \omega_k t + b_k \sin \omega_k t) \tag{1}$$

where $\omega_0 = 2\pi/T$, T is a period, and $\omega_k = k\omega_0$.

- The we obtain the coefficients:

a_k and b_k

$$a_k = \frac{2}{T} \int_0^T f(t) \cos \omega_k t dt \tag{2}$$

$$b_k = \frac{2}{T} \int_0^T f(t) \sin \omega_k t dt \tag{3}$$

Fourier Serise

- In the numerical calculations, we divide the period T into N discrete intervals:
  - $0, \Delta, 2\Delta, \cdots, (N-1)\Delta$
  - $T = N\Delta$
- Approximate a_k and b_k using

Approximation:

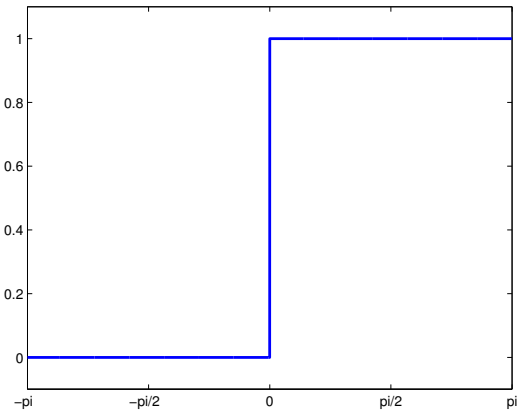
$$a_k \approx \frac{2\Delta}{T} \sum_{i=0}^{N-1} f(t_i) \cos \omega_k t_i \tag{4}$$

and

$$b_k \approx \frac{2\Delta}{T} \sum_{i=0}^{N-1} f(t_i) \sin \omega_k t_i \tag{5}$$

where $T = N\Delta$

Example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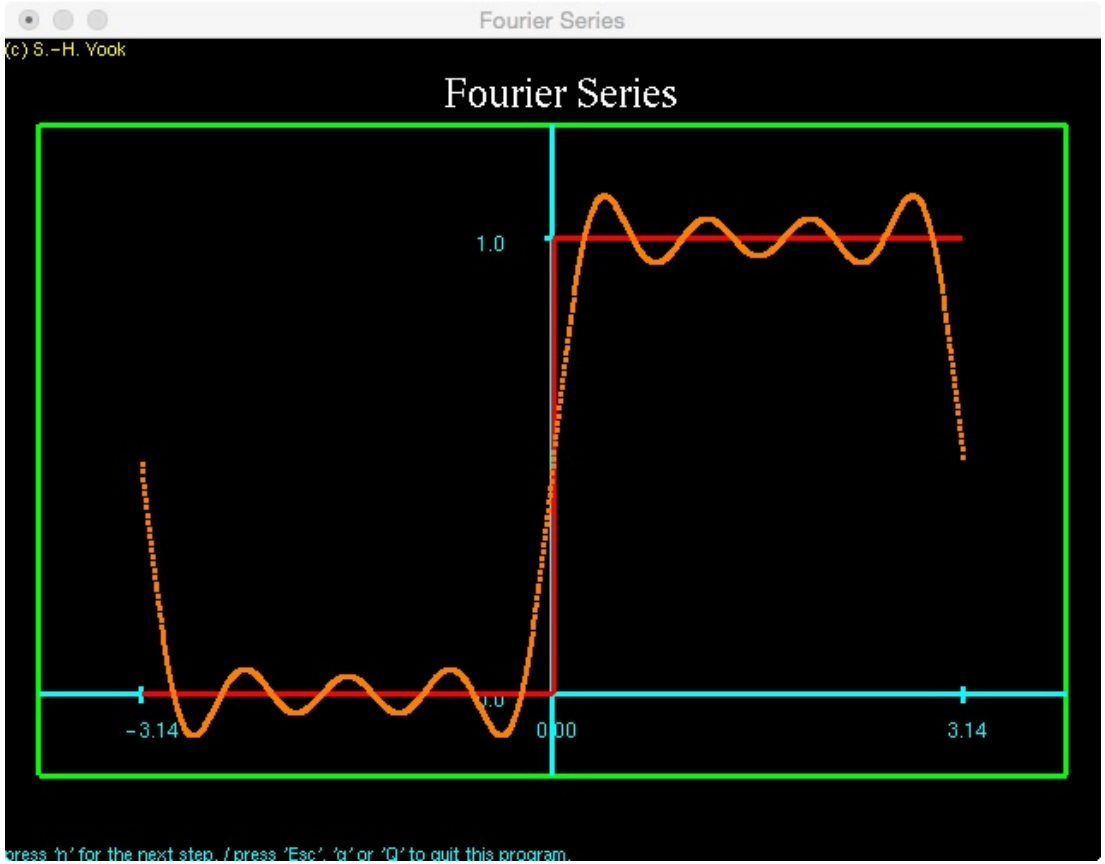
0.4
0.2
0
−pi
−pi/2
0
pi/2
pi


Fourier Series
(c) S.-H. Yook
Fourier Series
1.0
-3.14
0.00
3.14
press 'n' for the next step, / press 'Esc', 'q' or 'Q' to quit this program.