Lamb-lion Problem on Network structures

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- **lamb-lion problem**

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- **Tree Structure Scale-free**

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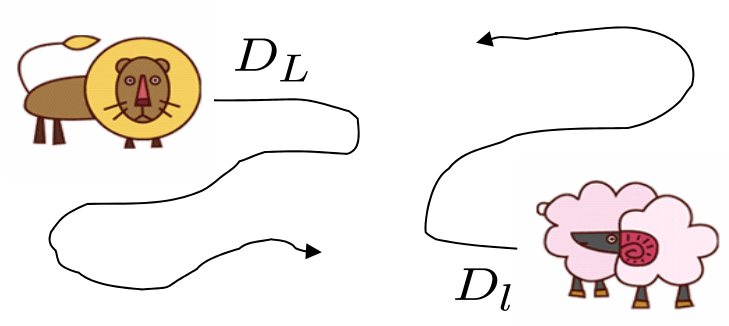
I. Introduction

lamb-lion problem

P.L.Krapivsky and S.Redner
J.Phys.A 29, 5347 (1996)



What is the survival probability $S(t)$ of a diffusing lamb which is hunted by N hungry lions?

$d \geq 3$ "unsuccessful" capture

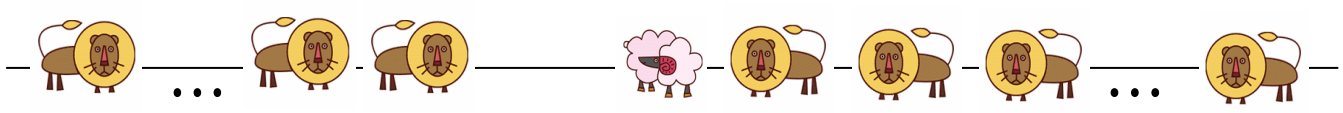
$$S(t) \sim finite$$

$d = 2$ "successful" capture

$$S(t) \sim (log\ t)^{-N}$$

(The lamb dies with probability one.
However, the average life time of the lamb is infinite!!)

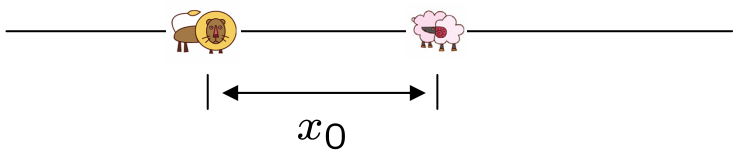
$d = 1$



N lions M lions

$S_{N,M}(t)$: The survival probability when initially N lions are placed to the left and M lions are placed to the right of the lamb.

one lion

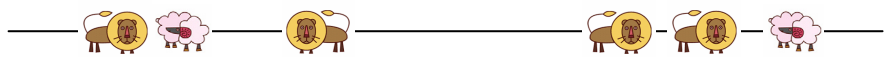


x_0

(the initial separation between the lion and the lamb)

$$S_{1,0}(t) = S_{0,1}(t) \sim \frac{x_0}{\sqrt{(D_L + D_l)\ t}}$$

two lion

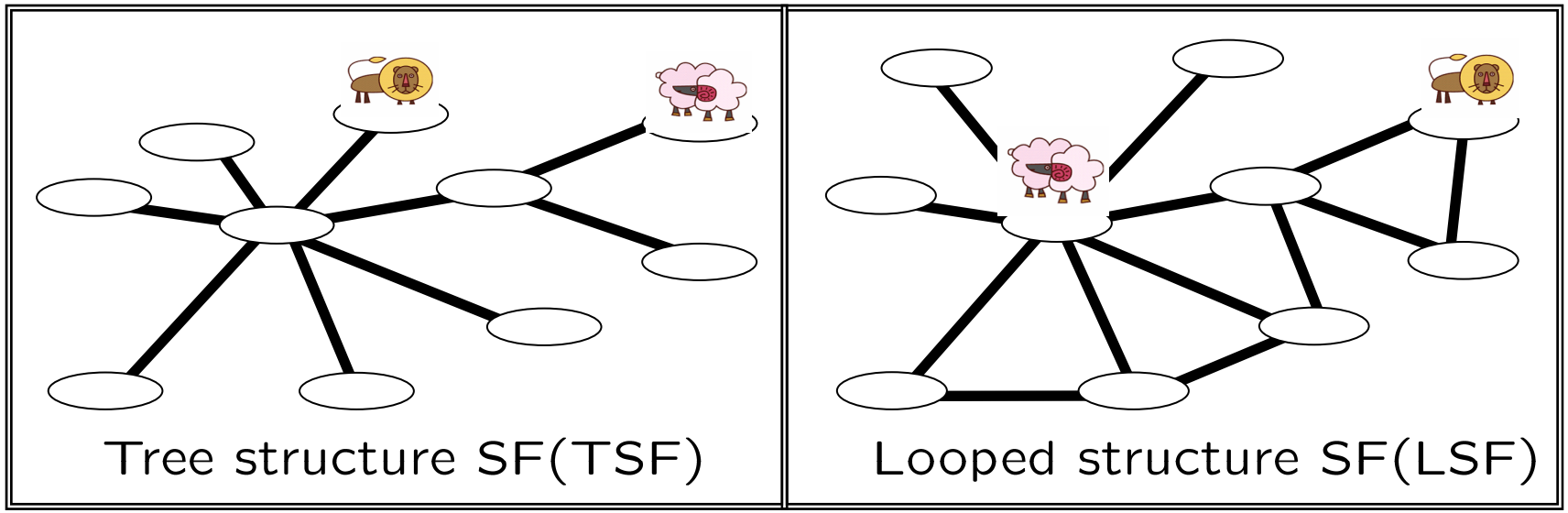


$$S_{1,1}(t) \sim t^{-\beta_{1,1}} \qquad S_{2,0}(t) \sim t^{-\beta_{2,0}}$$

$$\begin{cases} \beta_{1,1} \to \infty \\ \to 1 \\ = 3/2 \end{cases} \qquad \begin{cases} \beta_{2,0} = 1/2 & (R \to \infty) \\ = 1 & (R \to 0) \\ = 3/4 & (R = 1) \end{cases}$$

$$R = \frac{D_l}{D_L}$$

II. Model



- Condition
  - fixed network size N
  - one lion
  - $D_l = D_L$ $(R = 1)$
- Initial position
  - random node– random node
  - Hub – dangling
  - dangling – dangling
- Measurement

$S(t)$: the survival probability of the lamb

$< T >$: the average life time

III. Results

$$S(N,t) \sim S_\infty f(t/N^z) \qquad \begin{cases} f(x) = 1 & (t \ll N^z) \\ f(x) \to 0 & (t \gg N^z) \end{cases}$$

$$< T > \; = \int_0^\infty t(-\frac{dS(t)}{dt})dt \qquad (x = t/N^z)$$

- On LSF case

$\gamma \geq 3$ $\quad < T > \sim N^z$

$\gamma < 3$ $\quad < T > \overset{?}{\sim} N^z$ (Saturation)

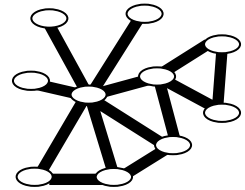
- On TSF case

$\gamma \geq 3$ $\quad < T > \sim N^z$

$\gamma < 3$ $\quad < T > \sim N^z$ $\quad S(t) = ?$

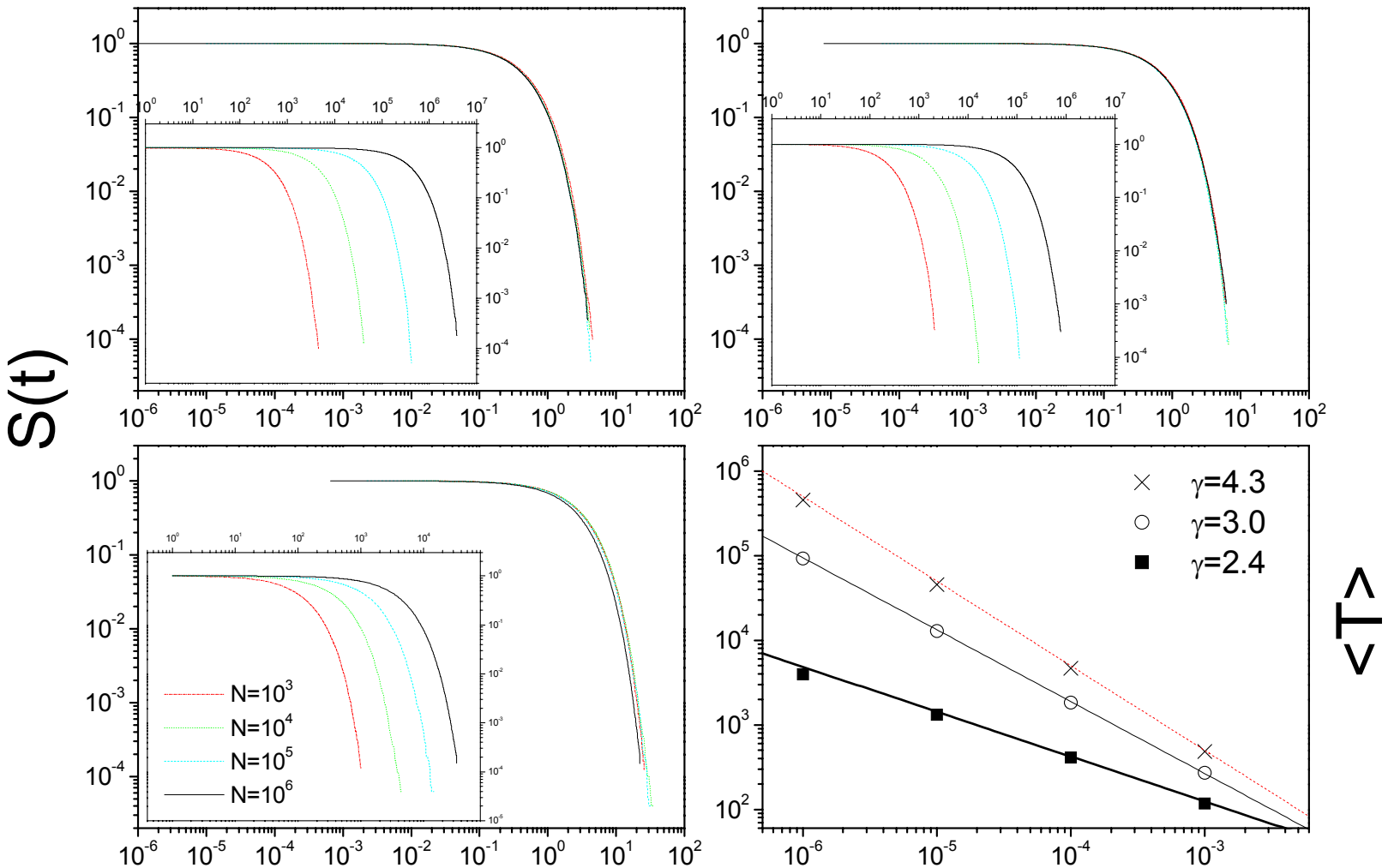
LSF / random node – random node

$N = 10^3 \sim 10^6$



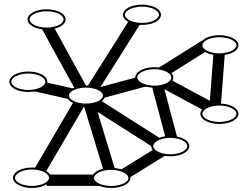
$\gamma = 4.3$ t/N

$\gamma = 3.0$ $t/N^{0.85}$



$\gamma = 2.4$ $t/N^{0.53}$

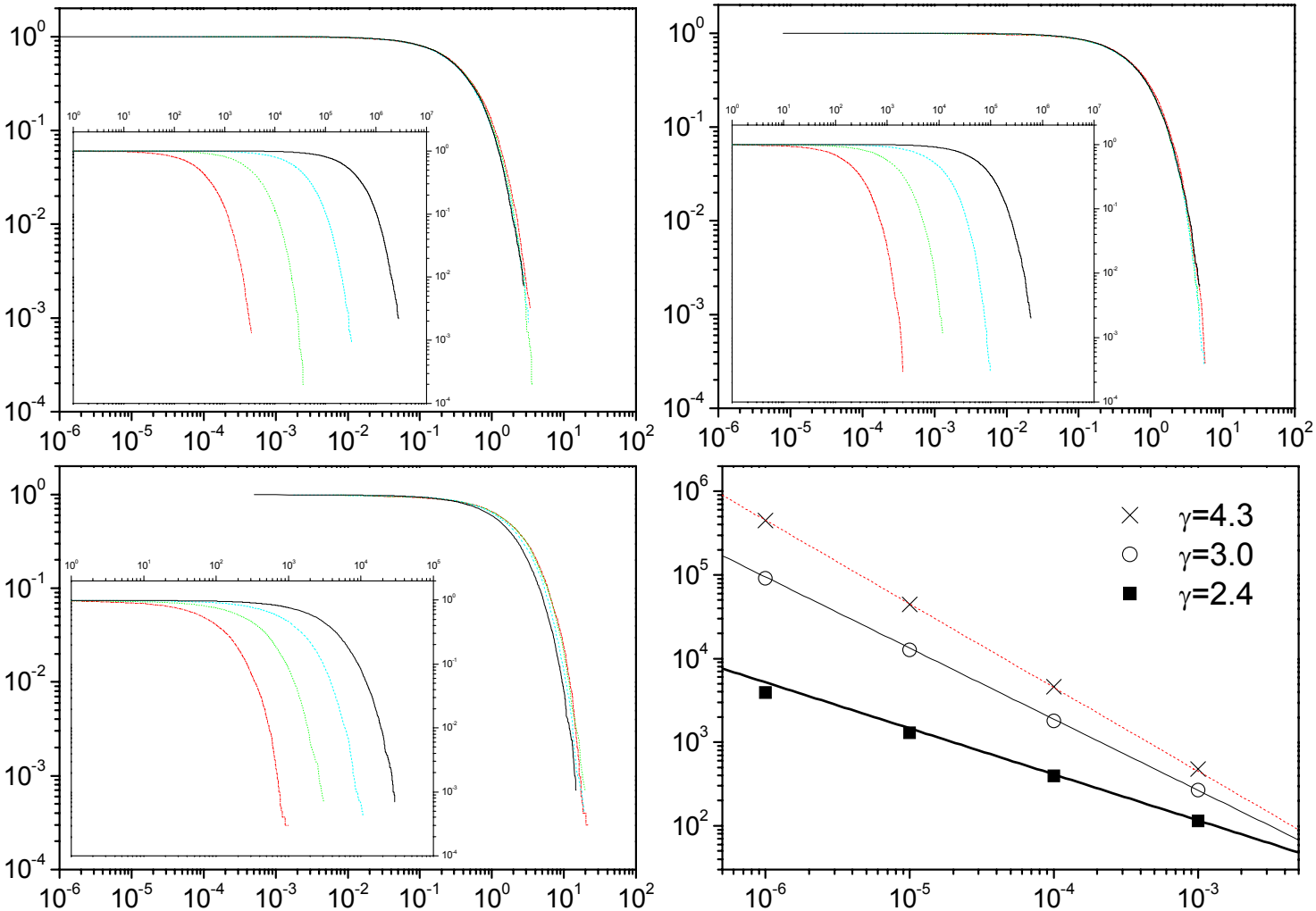
LSF / Hub – dangling



$\gamma = 4.3$ t/N

$\gamma = 3.0$ $t/N^{0.85}$

S(t)

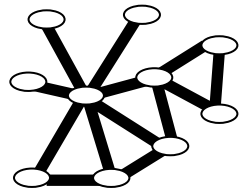


<T>

$\gamma = 2.4$ $t/N^{0.55}$

1/N

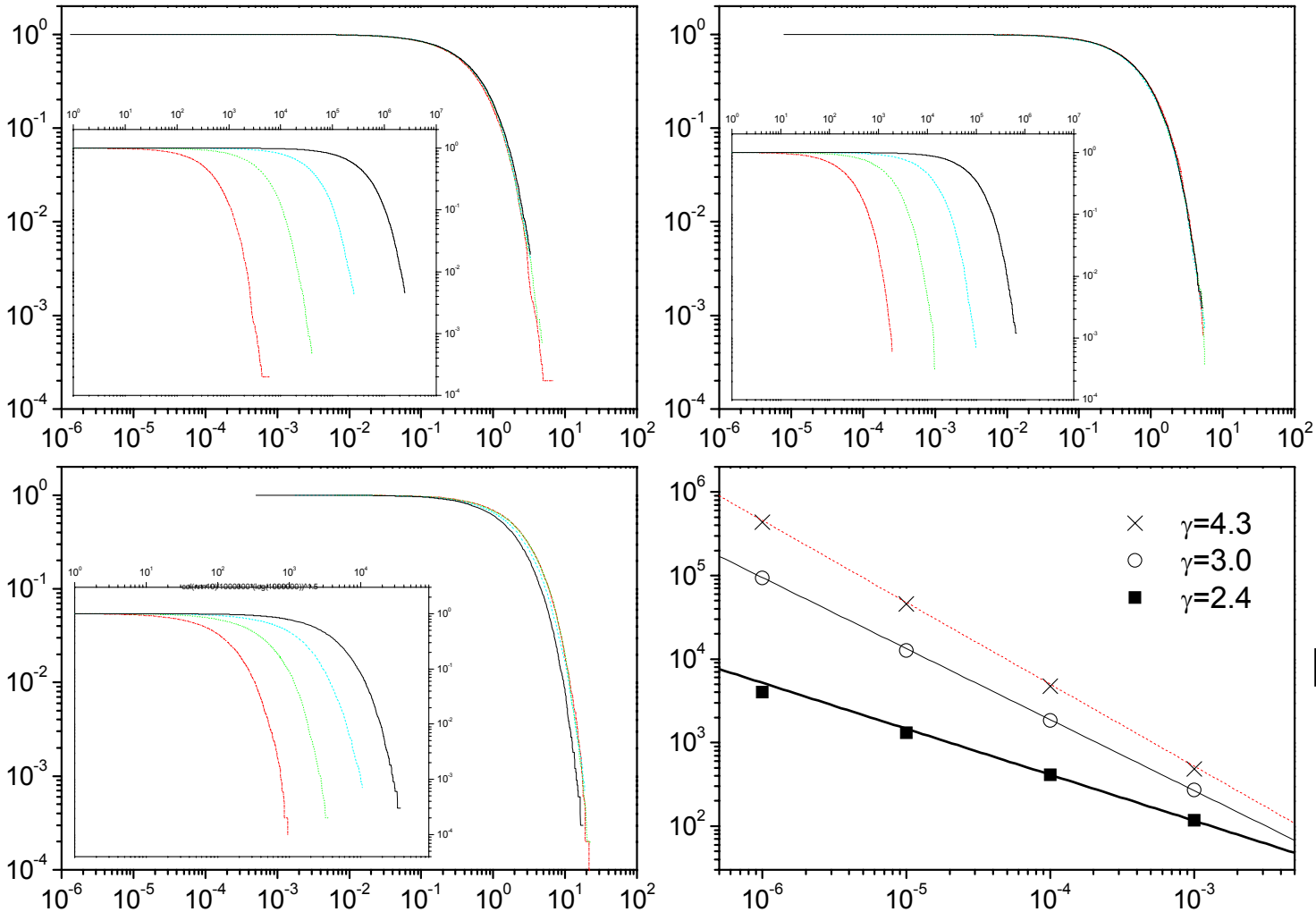
LSF / dangling – dangling



$\gamma = 4.3$ t/N

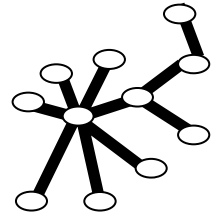
$\gamma = 3.0$ $t/N^{0.85}$

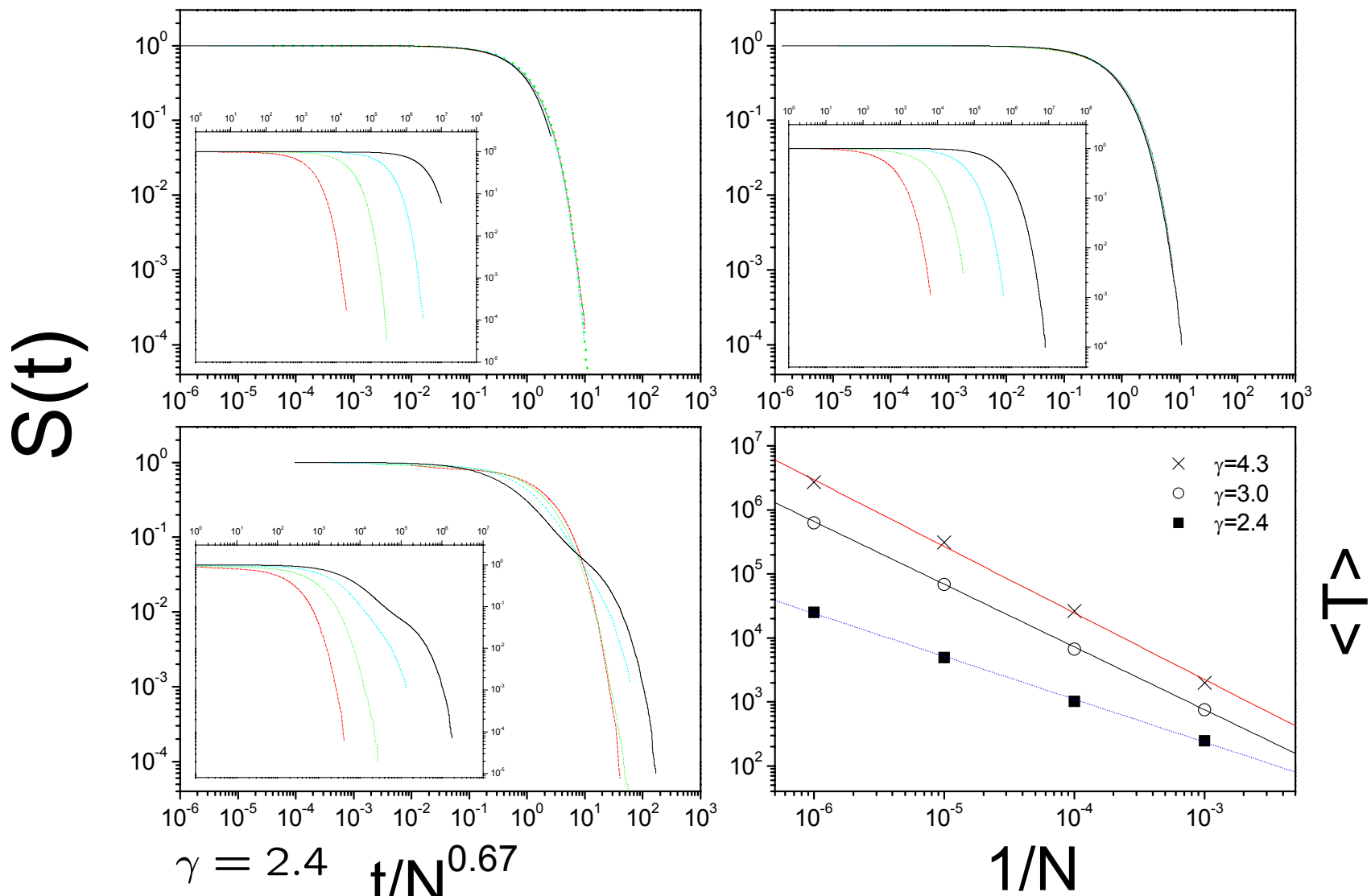
S(t)



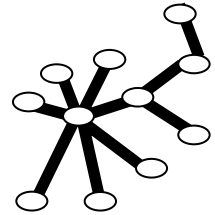
$\gamma = 2.4$ $t/N^{0.55}$

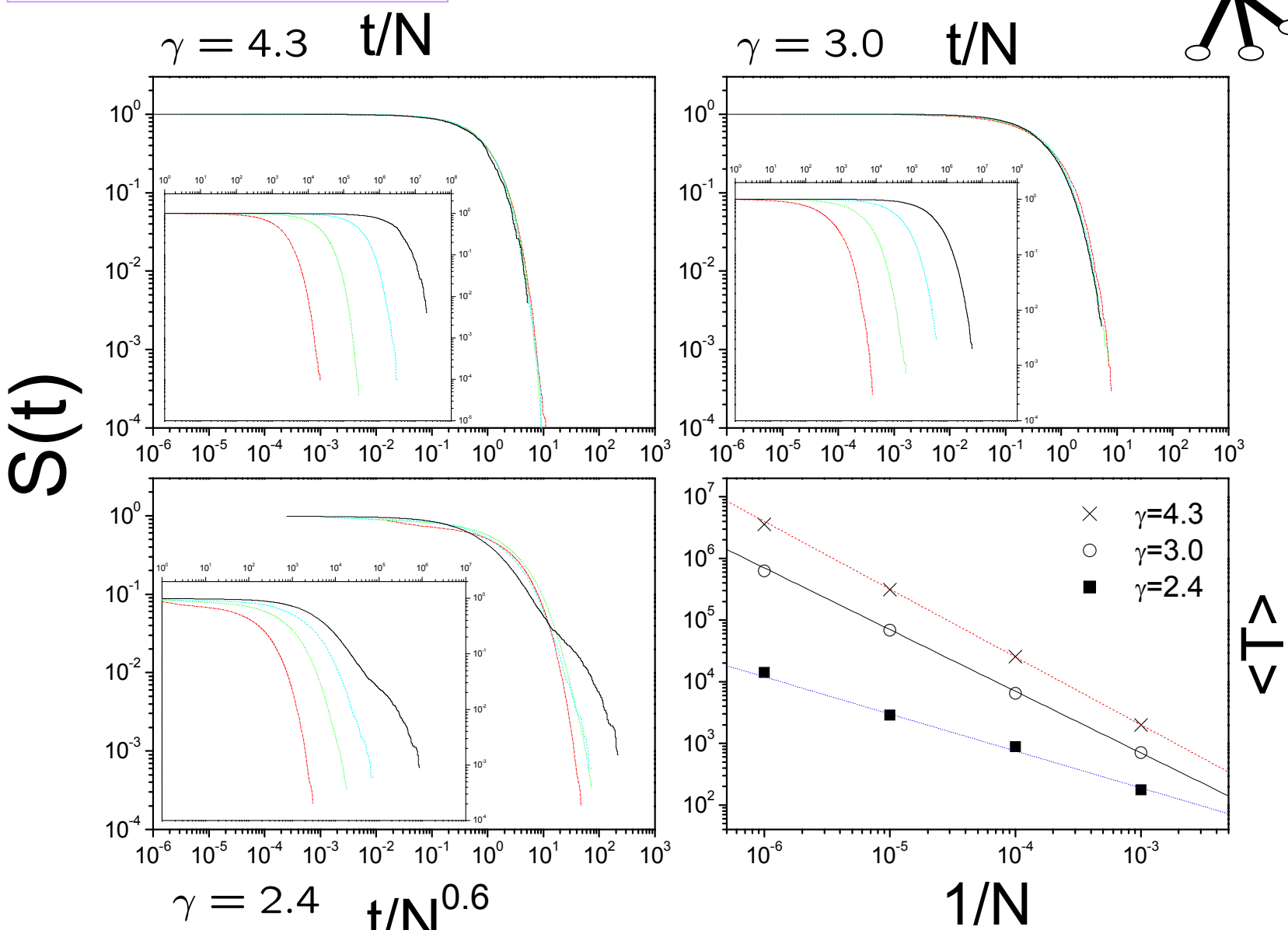
TSF / random node – random node



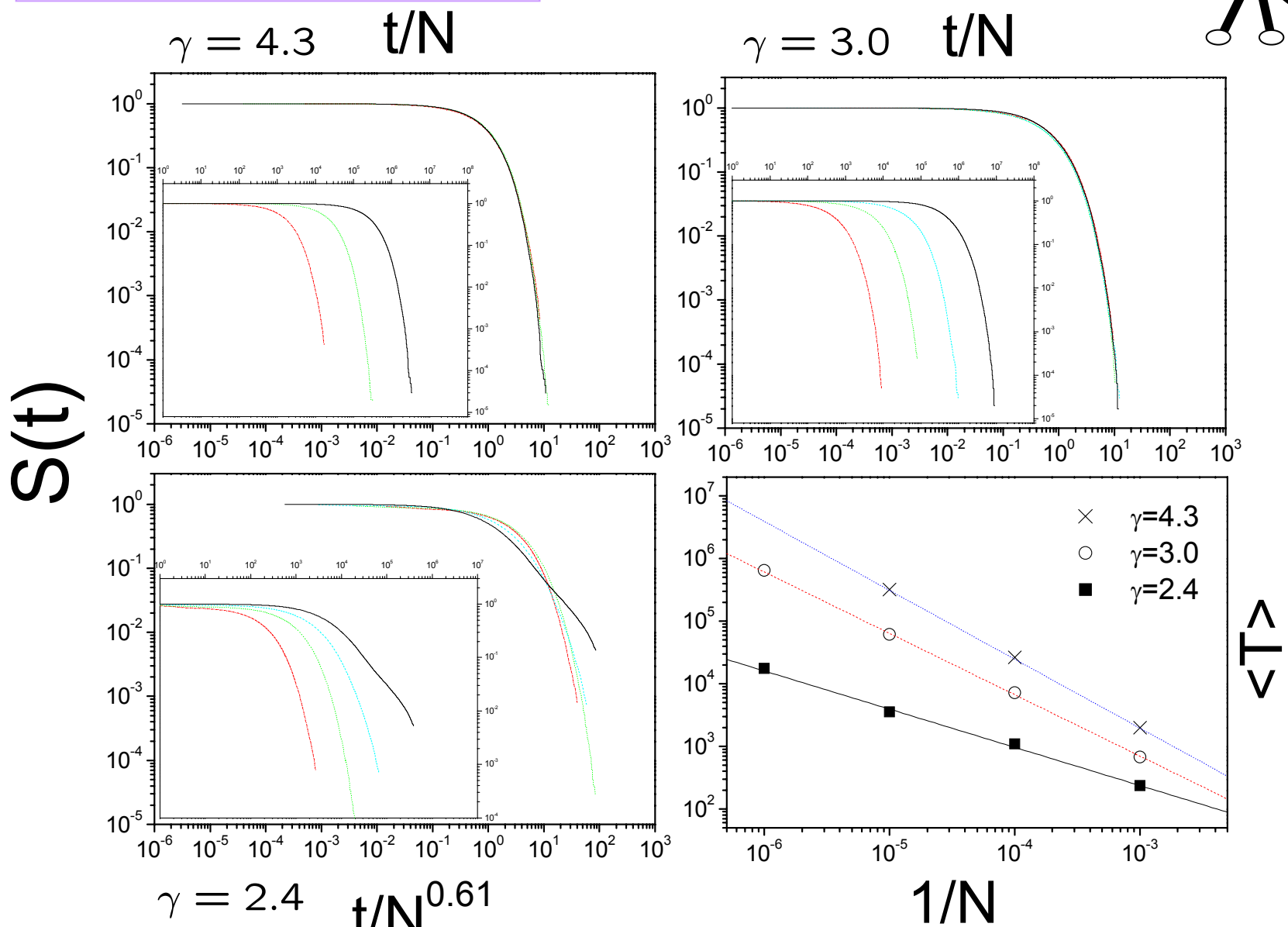


TSF / Hub – dangling





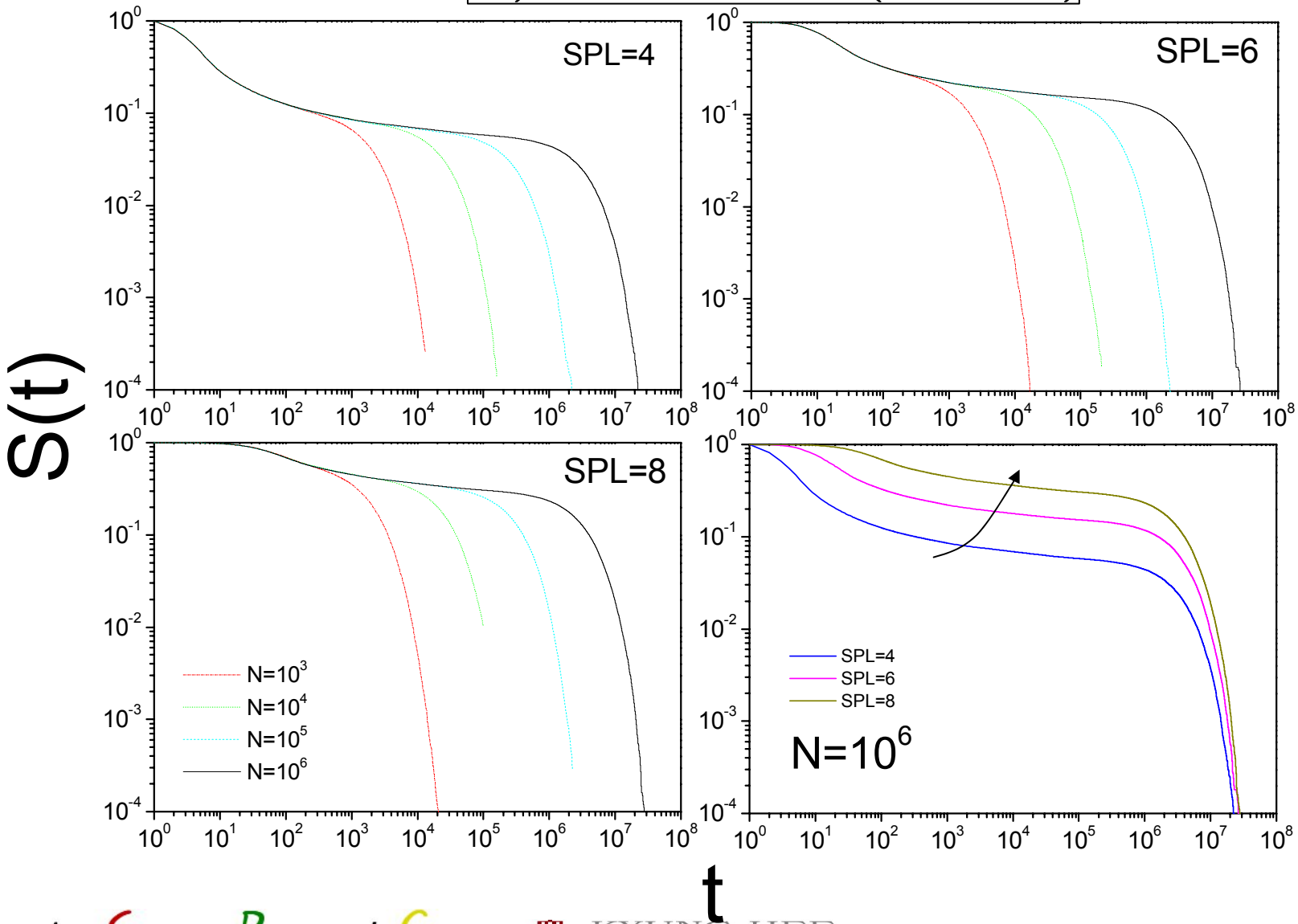
TSF / dangling – dangling



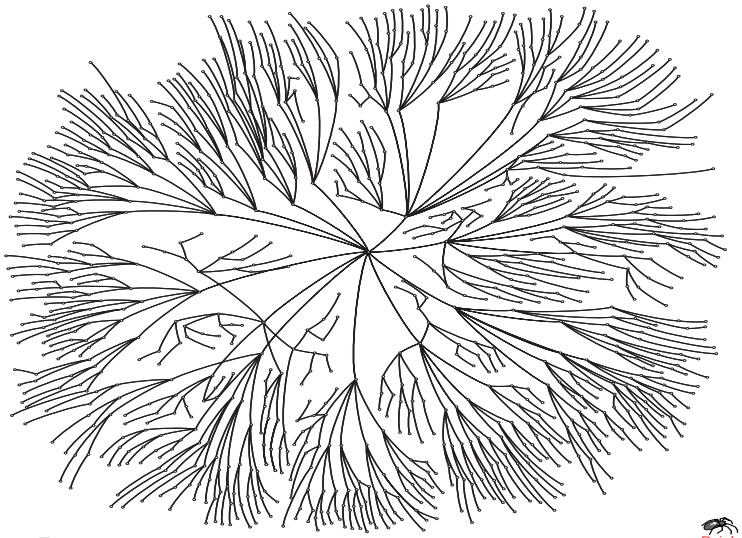
TSF / dangling – dangling (SPL = Shortest Path Length)

$\gamma = 4.3$ SF

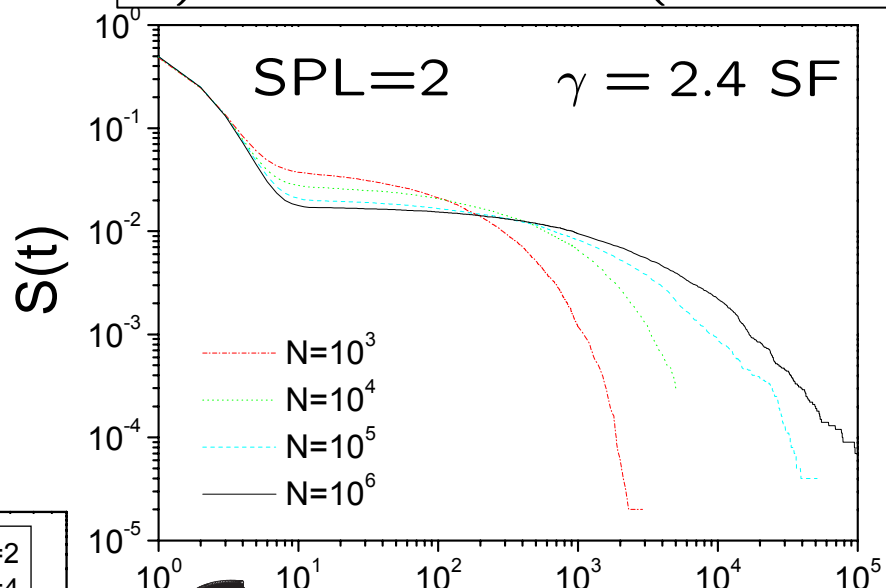
ex) diameter= 4.0 ($N = 10^3$)

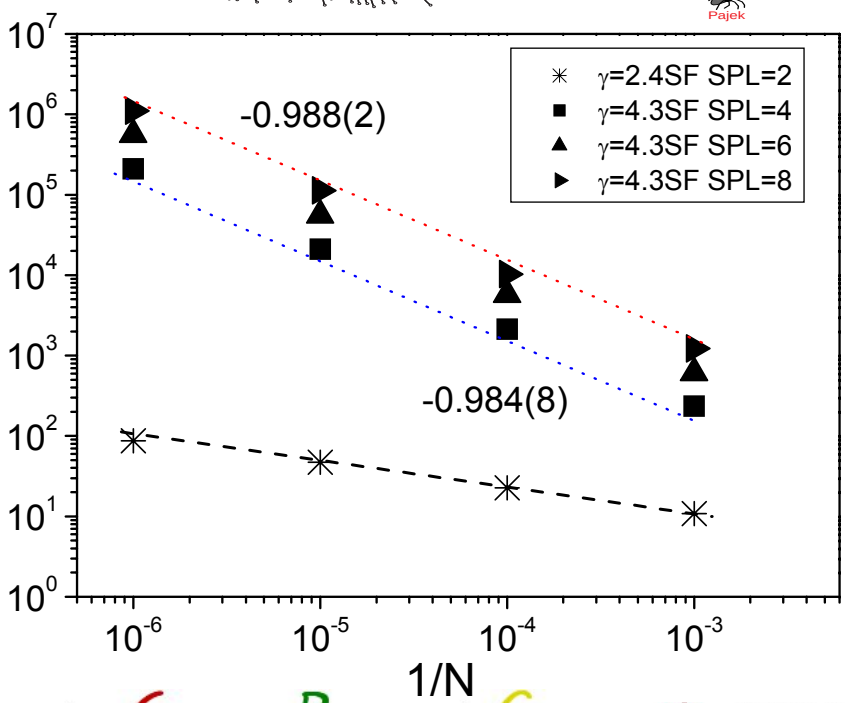


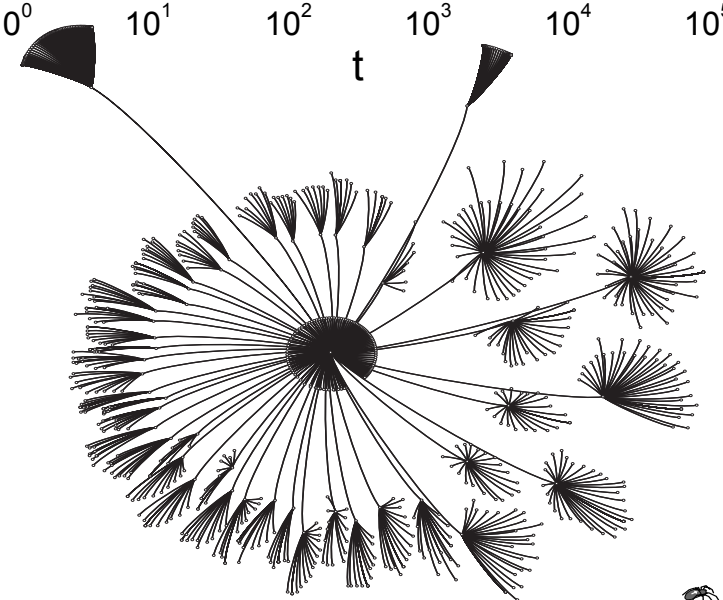
TSF / dangling – dangling(SPL = Shortest Path Length)



ex) diameter= 11.2 ($N = 10^3$)







IV. Summary

- We study the lamb-lion problem on network structures.
  Used networks are LSF and TSF.

- The results on $\gamma \geq 3$ LSF and TSF case

$$< T > \sim N^z$$

- The results on $\gamma < 3$ LSF case

$$< T > \overset{?}{\sim} N^z$$ (Saturation?)

- The results on $\gamma < 3$ TSF case

$$< T > \sim N^z \qquad S(t) = ?$$

Initial position
rand-rand
Hub-dangle
dangle-dangle

- We find that the survival probability is changed by
  the initial shortest path length between the lamb and the lion on LSF.